Equilibrium Displacement Models

Theory, Applications, & Policy Analysis

Gary W. Brester, Joseph A. Atwood,
& Michael A. Boland

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»Preface

Although equilibrium displacement models (EDMs) have a long and wide-ranging history in agricultural and resource economics research, the fundamental development of EDMs has not—to our knowledge—been formally published. Although EDMs are not the exclusive domain of any particular research institution, their development seems to be associated with specific university graduate programs. Many of those who develop and use EDMs have at least a tangential relationship with the Department of Agricultural and Resource Economics at North Carolina State University. Even researchers from that background tend to have been introduced to the modeling strategy through working papers and, in many cases, class notes rather than in any formal context.

Without question, Michael Wohlgenant and the late Bruce Gardner have been enormously influential. Those who have worked with either of these economists have extended their own ideas into the development of EDMs. While a complete list of those who have been influenced by that department is long and distinguished, a few of those who have used EDMs in their own research efforts and have a "direct" NC State influence include Julian Alston, Gary Brester, George Davis, John Freebairn, John Mullen, Richard Perrin, Nicholas Piggott, and Daniel Sumner. This network of economists has spread the modeling strategy to dozens, perhaps hundreds, of others at U.S. and international universities and government agencies.

This book originated because of a question that Glynn Tonsor asked Gary Brester about an EDM model that they had previously used for a research project. Specifically, Glynn noted a discrepancy in the model. We explain the issue in Chapter 4. Gary originally visited with Anton Bekkerman about the issue. When Anton was unable to resolve the problem in short order, Gary knew that he was in for a long haul. The problem was that Gary, among others, had used an EDM that was not theoretically consistent. Specifically, some EDMs are not homogeneous of degree 0 in both input and output prices. After much consultation with Joseph Atwood, Gary and Joe decided to investigate these discrepancies. That was over 8 years ago, and the basics of this book began while Gary was on sabbatical at Lincoln University in New Zealand.

Gary would like to thank Lincoln University's Department of Agribusiness and Commerce for providing office space and collegiality at the start of this project. The Department's faculty and staff were very kind and friendly. Gary greatly appreciates their personal and professional interests in this work. In addition, he thanks the staff at the Lincoln University Wellness Center for their daily interest in Gary and his wife, Colleen, as they frequented their gymnasium. While in Lincoln, various Lincoln University staff members did their best to try to teach Gary how to play squash. They were patient, kind, welcoming, and very good at the game. Unfortunately, Gary can't say the same about himself. Gary and Colleen's accommodation hosts, Chrissy and Tony Wyllie, and their extended family treated them like their own. Finally, Gary notes that his academic peers have often voiced their appreciation and indebtedness to their major advisors for various career accomplishments. His experience was no exception. While earning his master's degree at Montana State University, John Marsh was an outstanding mentor who had (and needed) the patience of Job. At North Carolina State University, Michael Wohlgenant took a Montana farm kid under his wing. Mike taught Gary a lot about research and EDMs, but Gary should have listened more carefully. Mike wrote the basic tenets of Gary's dissertation on a napkin during afternoon coffee at Ole Time Hot Dogs in 1988. Gary still has the napkin.

Much of the technical material presented in this book was initially developed and included in a packet of class notes by Joseph Atwood. Joe's initial efforts with respect to EDM modeling resulted from a project in which he and Glenn Helmers were examining the potential regulation of nitrogen fertilizer applications to feed grain production in Nebraska. Joe was struggling with modeling the market effects of a policy that that would result in both grain yield and quality changes. A colleague recommended that he explore the approaches presented in Bruce Gardner's book, *The Economics of Agricultural Policy*, and R.G.D. Allen's 1938 classic book on mathematical economics. The results of that exploration are presented in Appendix 4A.

Joe also contacted Bruce with several questions regarding EDMs. During one conversation, they discussed an error in Bruce's book. Ever gracious, Bruce encouraged Joe to publish the results of the Nebraska project using a theoretically consistent EDM and to note the book's error in the paper. During the article's review process, Joe was shown that equivalent results could be derived using a cost function and Shephard's Lemma. Several researchers have developed EDMs using that approach. After their initial discussions, Joe and Bruce continued a series of rewarding and exciting (at least for Joe) conversations on other research problems until Bruce's untimely passing. Joe will always treasure his conversations with Bruce, Glenn Helmers, and numerous others who brightened the world in which they walked. Joe especially wants to thank Glenn Helmers for his inspiring classes, mentoring, and discussions that excited Joe about the economic way of thinking.

Michael Boland became involved in this project because a graduate student, Andrew Keller, wanted to pursue his doctorate following the receipt of his agricultural law degree. Andrew wanted to learn how to conduct agricultural policy research. Mike and Metin Çakır decided that Andrew needed to learn how to build an EDM. One of his dissertation essays used an EDM to consider the impact of minimum wage legislation in agriculture. The project led to the development of a freshman seminar regarding agricultural economics policies contained in the Farm Bill that Mike teaches. Mike wants to thank Joe and Gary for their hospitality while they worked at Montana State University on this book.

Two accompanying Microsoft Excel workbooks are available from the University of Minnesota's print-on-demand website. One (EDM examples Chapters 4-8.xlsx) contains a tab for every example presented in this book through Chapter 8. The second (EDM examples Chapter 9.xlsm) contains the simulations used to produce the sensitivity analyses presented in Chapter 9.

Finally, Gary thanks Montana State University for supporting his sabbatical at Lincoln University. We also acknowledge the expertise of Kyle Brester, kylebrestergraphics, in designing the book cover and developing the book's figures. The combine pictured on the book's cover belongs to Kelly Brester (Gary's brother). In the picture, Gary is operating the combine in a field of malting barley — although a satellite is steering the machine.

The technical editing and patience of (two-time Jeopardy! winner) Amy Bekkerman at Precision Edits is much appreciated. Andrew Keller provided painstaking editorial help with Appendix 4A. We also acknowledge the formatting expertise of Susan Everson of the University of Minnesota's Library Publishing Services, from which this book is available in both print-on-demand and open access formats.

»Chapter One

EQUILIBRIUM DISPLACEMENT MODELS

Applied economists frequently use equilibrium displacement models (EDMs), also termed linear elasticity models, for policy analyses because they can be used to estimate changes in prices and quantities that result from exogenous economic or policy shocks. These models are also widely used to estimate changes in producer and consumer surplus caused by exogenous economic shocks and to quantify the short- and long-term impacts of a variety of economic and regulatory actions across multiple markets. Because complex interactions exist in many markets, EDMs provide a comprehensive approach to modeling changes in market equilibria. EDMs are particularly suited for evaluating vertical market relationships among input suppliers, processors, and consumers and horizontal market relationships among input suppliers, processors, importers, and exporters. Further, EDMs can be developed to simultaneously include both vertical and horizontal market relationships.

An Overview of Equilibrium Displacement Models

EDMs have had a long, prominent history in applied economic analyses because they provide both modeling flexibility and consistency with basic economic concepts. For example, EDMs are derived from, and represent a set of, comparative static results expressed in elasticity form (Wohlgenant, 2011). One major advantage of EDMs is that they allow researchers to use quantity and factor (i.e., input cost) shares to estimate the relative importance of various supply/demand shocks on price and quantity equilibria. Moreover, researchers can use elasticity estimates from existing research without needing to estimate large systems of supply and demand equations. This frees researchers from concerns related to equation functional forms, data availability, and other estimation issues.

Nonetheless, researchers can use various approaches to model policy and/or exogenous shocks on market equilibria. For example, comparative statics are often used to hypothesize directional changes in endogenous variables resulting from exogenous shocks. Computable general equilibrium (CGE) models represent a computational approach to estimating changes in economic systems. Agent-based modeling and system dynamics have been used to evaluate interactions among related markets. To some extent, one could argue that EDMs are more comprehensive than partial equilibrium modeling strategies but less comprehensive than

From: Student@UEconomics.edu
To: Professor Watson
Date: Sunday, 21 Sept 2021 at 11:01 p.m.
Subject: EDM Models

Dear Professor,

I am a little confused after reading my class notes on the first chapter and studying for the first quiz. Are EDM models the same as CGE models? I sent this note from the class email list as you requested so everyone sees the questions and answer.

Thanks!
Boris

From: Professor Watson
To: Student@UEconomics.edu
Date: Monday, 22 Sept 2021 at 8:32 a.m.
Re: EDM Models

Dear Class,

Computable general equilibrium, or CGE models, were briefly discussed in class and represent a computational approach to understanding changes in economic systems. I suggested in class that one could argue that EDMs are more comprehensive than partial equilibrium modeling strategies but less comprehensive than CGE approaches. CGE models are widely used to analyze trade flows between nations (as you can see at https://www.gtap.agecon.purdue.edu/models/current.asp) or to analyze environmental policy models (visit https://www.epa.gov/environmental-economics/cge-modeling-regulatory-analysis). Other models used to solve economic problems include agent-based modeling and system dynamics approaches. We are not going to discuss CGE models in class except that you should know they exist and if you continue into an agricultural economics doctoral program at a school such as Purdue University, you will learn more about them. You do not need to know anything about them for the quiz or any assignment or exam in this class. I hope this answers your question.

All the best,
Dr. Watson

CGE approaches. Much like other partial equilibrium approaches, EDMs are both less data intensive and computationally less expensive than CGE models.

Economists commonly use elasticities to understand complex market situations. Economic theory suggests that proportional or relative changes in economic factors are important predictors of human behavior. EDMs are intrinsically built on elasticity concepts and provide both modeling advantages and disadvantages. Advantages include the ability to quantify exogenous shocks and/or policy changes on market outcomes. Such models can be used to forecast outcomes based on comparative static projections. EDMs use linear approximations of supply and demand functions to evaluate shocks in a comparative static regime rather than the often difficult, if not impossible, simultaneous simulation solutions to nonlinear equations. This book shows the equivalency of comparative statics and EDM approximations for modeling exogenous shocks to systems of linear supply and demand functions. In addition, we illustrate the equivalency of EDMs and comparative statics when markets are characterized by nonlinear supply and demand functions. EDMs allow for systems of equations to be parameterized and calibrated using commonly available factor shares and elasticity estimates. These elasticities and factor shares can often be calculated or obtained from the extant literature. Further, economists can delineate short-run outcomes from long-run outcomes by considering constraints on adjustment processes. The easing of constraints over time is often modeled using more elastic or less inelastic own- and cross-price elasticities of supply and demand.

But EDMs, like all models, also have disadvantages. By construction, EDMs rely on linear approximations of what are almost certainly nonlinear functional forms of human behavior in both demand and supply equations. Consequently, EDM results are more reliable for relatively small exogenous shocks or policy changes unless the underlying demand and supply functions are only moderately nonlinear. In addition, researchers must decide which sectors of an economic system to include in EDMs, which necessarily means that some sectors will be ignored.

An important feature of economic analyses is the consideration of the unintended consequences of various regulatory and market interactions. Economic changes seldom occur in isolation because many markets are often interrelated. Hence, economic models should account for these interactions. Such efforts also force researchers to consider economic behavior explicitly and objectively in their models. EDMs are highly structured and transparent, which lends them to professional critique and replication and allows for a formal understanding of impacts among vertically or horizontally linked markets. In addition, such models can be used to impose theoretical economic principles, such as homogeneity in input and output prices in production processes.

Computational models can also be used for such purposes, but EDMs are more transparent and use basic linear algebra, which simplifies computational procedures. Hence, EDMs are more tractable than many other computational models, and error checking is more practical. Models that include well-specified nonlinear functions can be used to obtain equilibria changes caused by exogenous shocks. However, specifying nonlinear functional forms and computational solutions is often cumbersome. Further, it is often not obvious whether, or how, such models account for feedback effects, and using intuition to assess these effects can fail or be biased. For small changes, EDMs can approximate nonlinear behavior and provide results similar to those generated by more complicated computational approaches. The EDM approach allows researchers to construct proxies to structural models that explicitly consider, and allow for the testing of, market interactions, economic principles, and consumer and producer behavior.

Agricultural and resource regulatory policies are often applied directly to agricultural or extractive resource producers. However, modeling such policies is frequently intractable because production functions are either difficult or impossible to identify. EDMs allow these primal problems to be converted into the dual space using Lagrangian constrained optimization procedures. This allows economic problems to be specified as functions of more identifiable factors (e.g., factor shares, own- and cross-price demand and supply elasticities, and elasticities of input substitution) while considering horizontal and/or vertical market connections.

Researchers often use mathematical models rather than graphical analyses to evaluate the total impacts of complex economic interrelationships as the latter are often unable to evaluate relationships among markets. By construction, graphical analyses can only consider two variables while holding all other factors constant. Consequently, such approaches cannot evaluate the total-response changes that result from exogenous shocks because they do not account for feedback effects among markets. In addition, calculating changes in consumer surplus by integrating under a demand curve is incorrect if the approach uses a static demand function rather than a total-response demand function. EDMs consider feedback effects among markets that are included in the model using matrix algebra. Consequently, such models trace a linear path from an initial equilibrium point to a new equilibrium solution. These linear paths are termed equilibrium trajectories. In addition, EDMs can trace these trajectories using elasticities and factor share estimates. These outcomes are equivalent to those obtained from comparative static approaches because EDMs are developed using simple row operations of the total differentials of supply and demand equations. Thus, depending upon the problem's dimensionality, EDMs approximate equilibria responses along tangent lines, planes, or hyperplanes.

EDMs can account for multiple market interactions caused by exogenous shocks. When the degree of curvature of demand and supply functions is unknown, EDMs should only be used to estimate changes in endogenous variables resulting from relatively small shocks. Economists are most often interested in relatively small perturbations to economic activity. And, many economic shocks and policy changes are sector specific. Thus, EDMs provide estimates of changes in equilibria for the type of shocks and policy changes most frequently encountered. In recent years, attempts have been made to provide confidence intervals for predicted changes in market equilibria obtained from EDMs. Confidence intervals can be produced by EDMs if the variances of and, when appropriate, correlations among elasticities can be obtained or estimated.

Although the primary purpose of this book is to develop and illustrate EDM research applications, we find that these models help economics students better understand market interactions. In addition, the often-imposed holding-all-else-constant assumptions of partial equilibrium models can be relaxed with EDM approaches. EDMs consider the complex interactions of multiple markets that occur in virtually every sector of an economy.

EDM Applications

Markets seldom function in isolation from one another. Some markets are related through vertical relationships. This is especially the case when one sector's output is used as another's input. Other markets have horizontal relationships in the sense that the production of some good or service often requires the use of several related inputs. For example, a sector's total supply may come from multiple sources, such as domestic production and imports. A soybean processor not only requires soybeans to produce oil and meal but also demands labor, energy, and capital inputs. In addition, the effects of an exogenous shock in one market not only influence equilibria outcomes in others but also changes consumer and producer surplus. For example, exogenous shocks to the corn production sector can influence soybean processors because the two commodities are production substi-

Why Is This Book Needed?

This book fills a void in both the theoretical and applied development of EDMs. We begin by illustrating the relationships between differential calculus, perturbation theory, comparative statics, and equilibrium conditions. The relationship between linear approximations of nonlinear functions and comparative statics as well as necessary linear algebra and differentiation techniques are presented. We then apply these tools to linear approximations of economic models, beginning with a single equation and then expanding to include multiple-equation models that represent both vertically and horizontally related markets. Several approaches to estimating changes in producer and consumer surplus that result from exogenous shocks as well as a method for hypothesis testing and sensitivity analyses of EDM estimates are developed. Throughout this book, we highlight how these models can be used for policy analyses and for predicting the effects of exogenous perturbations on endogenous market equilibria.

tutes. Further, the process of modeling interrelated markets often involves a variety of problematic estimation issues.

Vertical Relationships

Vertical market relationships exist among most agricultural and food processing sectors. For example, producers of beef cattle genetics sell their products to cow-calf producers, who in turn produce calves. Calves are then either sold to backgrounders or directly to feedlot operators, who produce fed cattle. Fed cattle are used as an input by meat processors to produce a variety of beef products, which are then sold to hotels, restaurants, and institutional outlets as well as retail grocery and meat stores. An exogenous shock at any of these levels has implications for markets both upstream and downstream of the initially affected market. EDMs are particularly suited for measuring the effects on market equilibria along vertical supply chains as well as for estimating changes in producer and consumer surplus.

To illustrate, consider a simple market representation of the U.S. beef cattle sector. Figure 1.1 presents total-response supply and demand functions for two vertically related markets: fed cattle production and retail beef consumers. Feedlot producers supply fed cattle, represented by a primary supply curve S_0^F, and food processors have a derived demand for cattle, represented by D_0^F.[1] Cattle processors convert fed cattle into beef products, which are supplied in a variety of forms to the food service and retail sectors, represented by the derived supply curve S_0^R. Ultimately, consumers demand beef products in a variety of forms, represented by the primary demand curve D_0^R.

The price, p_0^F, represents the initial equilibrium price of fed cattle, and p_0^R represents the initial equilibrium retail price of beef. The initial equilib-

1 Throughout this chapter, superscripts denote a sector and subscripts denote equilibrium points.

Vertical and Horizontal Linkages in EDMs

This chapter uses an example from beef markets to demonstrate vertical and horizontal linkages. EDMs have been most often used to study markets with vertical and/or horizontal linkages, such as the meat processing sector, in which vertical markets separate animal production from consumer products. In addition, several vertical sectors exist within the beef value chain including cow/calf operations, backgrounding or stocker producers, and fed cattle production. Retail beef demand is also influenced by the horizontal markets for beef substitutes such as pork and poultry. In addition, market power can be introduced in these models to denote monopolistic or oligopolistic structures. Most agricultural markets also include vertical linkages, such as in the production of soil nutrients: vertical linkages with inorganic, dry and liquid synthetic fertilizer, and organic fertilizers such as animal manure as well as horizontal linkages with other fertilizer substitutes.

rium quantity, q_0, represents retail-weight equivalents at the consumer level translated to the quantity of fed beef cattle required to produce that quantity. The example abstracts from inventories of beef cold-storage inventories.

Consider the impact of an exogenous shock to the primary demand for beef products (perhaps from a reduction in per capita income) that reduces demand to D_1^R (Figure 1.2). The shift in demand reduces the retail equilibrium price to p_1^R, the farm-level equilibrium price for fed cattle to p_1^F, and the market equilibrium quantity to q_1. The final equilibria, however, involves a dynamic feedback relationship between the two markets. Hence, EDMs are suited to obtaining new equilibria while incorporating feedback effects.

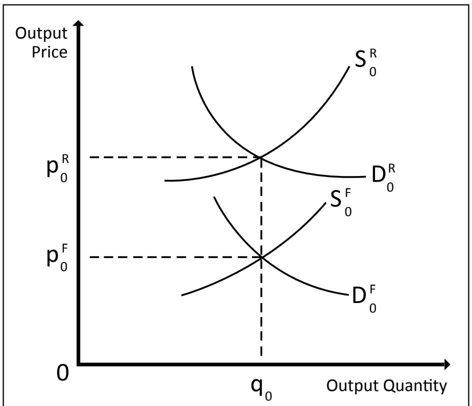

Figure 1.1. Market Equilibrium for Two Vertically Related Markets

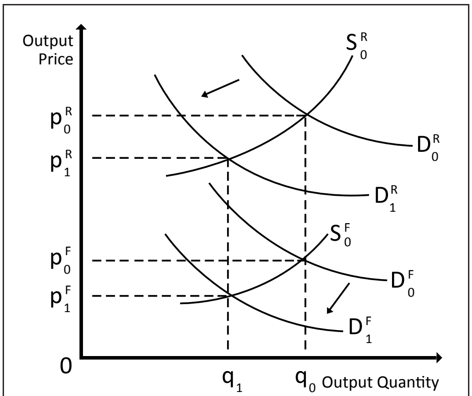

Figure 1.2. Changes in Market Equilibria for Two Vertically Related Markets

Horizontal Relationships

Consumer beef demand is influenced by a variety of factors, including the prices of substitute meat products. Consequently, horizontal market relationships exist across retail meat species and many other products. Horizontal linkages also occur on the supply side of most markets. For example, retailers supply beef products to consumers through purchases of meat products produced by food processors. However, retailers also purchase other inputs—such as labor, energy, and capital—and beef processors may acquire beef inputs from domestic or foreign sources and use many other inputs in production processes. Hence, total demand for a given input may encompass two or more production sectors. EDMs can incorporate horizontal linkages among input and output markets.

Surplus Measures

EDMs have been widely used to estimate changes in consumer surplus, producer surplus, and deadweight losses resulting from exogenous economic or policy shocks. However, care must be exercised when using EDMs for this purpose. In the absence of knowing the actual underlying demand and supply functions, surplus changes cannot be calculated relative to a total surplus measure. Hence, such changes should include an appropriate scale for comparisons, such as the size or total revenue of an industry. In addition, changes in surplus can only be calculated for exogenous shocks to behavioral equations or policy-induced price or quantity wedges. That is, EDMs should not be used to evaluate changes in consumer or producer surplus that result from exogenous shocks occurring beyond the behavioral equations included in the model.

In any partial equilibrium analysis, researchers must decide which components of an economic sector are to be treated as exogenous versus endogenous to the system. EDMs specify the endogenous elements of a system in a general form and then consider the approximate effects of exogenous shocks. In general, EDMs are unable to evaluate changes in surplus of exogenous shocks emanating from sectors that are not explicitly included as behavioral equations in the model. On the other hand, if an exogenous shock occurs to a behavioral equation within an EDM, changes in producer and consumer surplus can be estimated. In addition, these estimates encompass feedback effects among the markets included in the model specification.

Estimation Issues

Comparative static analyses of exogenous shocks on related markets require the use of various supply and demand or, in some cases, reduced-form equations. The complexity of estimating such functions expands as additional related markets are included. For example, estimating supply and demand functions for a market from a single set of price and quantity data is likely to incur identification issues. Further, estimating supply and demand functions of related markets will not only suffer from the same issues but may complicate the estimation of all equations if error structures are related.

EDMs allow researchers to use supply, demand, and input substitution elasticity estimates from the extant literature. On the other hand, if a researcher wants to construct confidence intervals for estimates of new market equilibria and surplus changes, then variances and covariances of elasticity estimates are needed. These can sometimes be obtained from the existing literature, but more often they can only be obtained by estimating systems of equations as part of the modeling exercise.

Summary

Equilibrium displacement models are constructed as linear approximations of unknown underlying and (probably) nonlinear demand and supply functions. An EDM is essentially a system of total differentials. EDMs constructed in the dual space allow them to be parameterized using estimates of supply, demand, and input substitution elasticities along with factor shares. This approach is more practical than estimating production and demand functions in the primal space. EDMs are much easier to construct, implement, and error check than most other partial and general equilibrium models. In addition, it is much easier and more tractable to directly incorporate various aspects of consumer and producer behavior into EDMs while maintaining theoretic consistency.

EDMs are flexible and can be applied to multiple vertical and/or horizontal markets. Nonetheless, they represent partial equilibria analyses and should only be used to quantify the effects on market equilibria for relatively small exogenous economic or policy shocks unless the underlying demand and supply curves are assumed to be only moderately nonlinear. The models have broad applications and are among the most often used to evaluate the effects of exogenous shocks on market participants. They are used to estimate the impact of exogenous shocks on producer and consumer surplus in addition to changes in price and quantity equilibria. Properly constructed, the models incorporate internal feedback effects of exogenous shocks.

Although economists frequently use EDMs, no single source exists that presents the theoretical and mathematical development of EDMs. In addition, many EDMs used in research efforts fail to correspond with consumer and producer economic theory, which can introduce bias into research results. This book explains the theoretical, mathematical, and practical aspects of developing EDMs. In addition, we show how EDMs can be used to explicitly include common economic theory regarding consumer and producer behavior, which provides a useful educational aid for those studying economics.

Who Will Use This Textbook?

This book provides the background and methodology needed to develop and implement EDMs. EDMs overcome the primary problem of using underlying, but unobservable, production functions because they are based on a dual approach which obviates the need to estimate production functions. The modeling strategy is motivated in terms of both policy analyses and exogenous shocks to various economic sectors. The book helps students understand and visualize interactions among market sectors. It is written for use in a graduate course in agricultural, resource, or applied economics with a focus on policy, production, or consumer economics. It fills a void in the literature as some EDM concepts are often only mentioned in passing because of space limitations in published journal articles or, more often, in copies of instructors' notes. This textbook comprehensively explains the development and use of EDMs.

»Chapter Two

EQUILIBRIUM DISPLACEMENT MODEL APPLICATIONS

Equilibrium displacement models (EDMs) are widely used in economics research, and their flexibility as a modeling strategy is evident by the broad array of issues addressed. EDMs have been used to evaluate exogenous shocks on the demand for inputs and to quantify changes in market outcomes that result from a host of market interventions such as trade, policy, and tax legislation. The modeling approach has been used to evaluate the impacts of technological change on market equilibria, the impacts of research and development investments, and the effectiveness of commodity advertising programs. Further, EDMs have found widespread use as a tool for evaluating the impact of market power and farm-retail price spreads on price and quantity equilibria. They have been extensively used to estimate changes in consumer and producer surplus that result from market shocks and legislative policies. Finally, more recent innovations include evaluating the precision of EDM estimates. This chapter provides a cursory review of research that use EDMs.

Input Demand

EDMs are rooted in the concept of elasticities, which were first expressed in quantitative terms by Allen (1938) and, later, Hicks (1957) with respect to an industry's derived demand for an input. Buse (1958) introduced the concept of total elasticity to agricultural economics, which provides an avenue and rationale for considering feedback effects generated by exogenous shocks. Muth (1964) was the first to consider reduced forms of a system of supply and demand functions for a single product using two factors of production and exogenous supply and demand shifters. Muth's application offers an early use of EDMs as a means for deriving supply and demand equations using general equilibrium models. Based on a set of six generalized equations, Muth obtained reduced-form equations that allowed for the development of industry supply and input demand functions. Using these derived demand and supply equations, Muth showed how changes in output price affect input demands, firm size, industry size, and total output. Further, he showed that changes in supply factors, substitution effects, and technological change can lead to indeterminate signs for other inputs by applying these concepts to the economics of housing and urban land use. Using previously estimated elasticities for housing availability, land availability, and substitutability, Muth derived elasticities for housing supply and land rents in urban areas. His model also illustrated the impact of urban growth and the differences between population growth in urban centers and surrounding suburban areas.

Wohlgenant (1989) constructed an EDM to test for the existence of constant returns-to-scale production functions and estimate reduced-form retail and farm prices for eight U.S. agricultural commodities. His results confirmed the importance of including elasticities of input substitution in such models and demonstrated the deficiencies of traditional methodologies for estimating retail-farm price linkages and derived demand elasticities. Wohlgenant concluded that allowing for input substitutability is important when modeling shocks to agricultural markets. Further, the traditional method of multiplying primary demand and price transmission elasticities to estimate derived demand elasticities may substantially underestimate the latter.

Market Intervention

Perrin and Scobie (1981) developed an EDM to determine the costs of government policies designed to increase caloric intake of low-income residents in Colombia, South America. Their model considers the effects of policies that shift supply and demand or create market price/quantity wedges. The generalized model was derived for multiple goods and supply functions and used elasticities to identify changes in equilibria. The authors addressed various program implications and potential issues related to black-market activities and socioeconomic status. They estimated the impacts of government programs on these groups with and without allowing for arbitrage. Their EDM was used to estimate the costs of subsidies and income transfers and the equilibria impacts on major agricultural products using price and income elasticity estimates.

Atwood and Helmers (1998) used an EDM to estimate the impact of proposed restrictions on the timing and level of nitrogen application on feed grain production. Reduced nitrogen availability harms both feed grain yields and quality. They estimated changes in surplus that occur for two types of proposed legislative actions and found that if a fertilizer tax or a per acre use restriction is imposed, social costs are underestimated if feed grain quality effects are ignored. Conversely, a restriction on the total amount of fertilizer use will overestimate social costs if the effects on grain quality are ignored.

Technological Change and Research and Development

Beginning with the use of an EDM by Freebairn, Davis, and Edwards (1982), researchers have contributed to a considerable literature regarding the distributional effects of research benefits. Alston and Scobie (1983), in commenting on Freebairn, Davis, and Edwards (1982), used the Muth (1964) model to show that the distribution of research benefits is sensitive to the degree of substitutability between farm and marketing inputs. Lemieux and Wohlgenant (1989) used an EDM to forecast the effects of biotechnological research expenditures on consumer and producer surplus. The authors used a linear programming model and experimental data to construct an EDM that included substitute goods and international trade in the U.S. pork sector.

Mullen, Alston, and Wohlgenant (1989) showed that significant input substitution occurs between raw wool and other inputs used to produce wool tops. The authors considered the Australian wool industry to quantify the effects of research funding directed at various production inputs. Building on the EDM research of Freebairn, Davis and Edwards (1982) and Alston and Scobie (1983), they examined how input substitutability affects returns across input markets using an EDM that included substitution between domestic and international wool. The authors found that input suppliers receive equal benefits regardless of where research funding is directed when input substitutability does not occur. In the presence of input substitutability, they found that marginal returns to research directed to wool growers would be greater than those generated by research directed to off-farm wool research. They also evaluated the incidence of a wool tax levied on farmers used to fund such research efforts and found that farmers bore the largest incidence of the tax because of input substitutability.

Holloway (1989) added to Muth's (1964) single-stage model with a two-stage marketing-sector that incorporated agricultural research gains. Holloway showed that if different elasticities of input substitution exist across marketing levels, then the impact of research on agricultural producers depends upon the marketing level to which the research is targeted.

Pendell et al. (2013) used an EDM to analyze the economic impacts of changes in beef cattle age and source verification requirements, the costs of implementing such technologies, and associated adjustments in the international trade of U.S. beef. They developed a multi-market EDM of the U.S. meat industry to estimate the effects of industry costs incurred through the adoption of age and source verification technologies on U.S. livestock and meat producers and consumers. They found that increased costs are dispersed throughout the vertically related marketing chain. Thus, such programs affect equilibrium livestock and meat prices as well as quantities exchanged in the market. The model recognizes that changes in retail prices for one meat commodity influence the demand for substitute meat products and that the adoption of age and source verification technologies could also positively influence domestic and international demand for meat products.

Zhao et al. (2000b) developed a large, multiple-sector EDM for the Australian beef industry to examine the effects of 12 research investment scenarios across the production, processing, and marketing value chain. The model included multiple vertical and horizontal markets. They found that producers and domestic consumers receive larger benefits from on-farm relative to off-farm research expenditures. The two groups also receive larger benefits from export marketing and promotion expenditures relative to domestic marketing and promotion expenditures.

Advertising and Product Attributes

Mullen and Alston (1994) used Lemieux and Wohlgenant's (1989) approach to measure the impact of leaner lamb meat on Australian lamb demand. They developed an EDM to estimate gains in producer and consumer surplus caused by a shift from a traditional to a leaner premium lamb product. The authors estimated simultaneous price and quantity changes for producers, processors, and consumers, combining exogenous product mix changes with changes in technology needed to meet processing standards for this new product. They considered the benefits of technological change between similar and/or substitutable products at the producer, processor, and consumer levels.

Lusk and Anderson (2004) used an EDM to estimate the effects of country-of-origin labeling laws on consumers, producers, and marketers. The authors constructed a model to estimate surplus effects using selected input substitution and demand elasticities. They included beef, pork, and poultry markets and cross-price elasticities under both trade and no-trade conditions. Their EDM was parameterized using elasticity estimates from the extant literature and demonstrated that consumers and producers do not share the costs of country-of-origin labeling equally.

Wohlgenant (1993) developed an EDM to quantify the effects of generic advertising on agricultural producers and found that the distribution of gains from advertising in the U.S. beef and pork industries is highly sensitive to the degree of input substitutability in the processing sector. Coupled with empirical results showing economically significant input substitution, he found that industry groups should consider reallocating funds away from advertising and off-farm research toward on-farm research.

Piggott, Piggott, and Wright (1995) showed how commodity interrelationships and advertising could be modeled with an EDM framework. Kinnucan (1999) extended an EDM to consider the effects of advertising while incorporating trade.

Cranfield (2002) found an inverse relationship between optimal advertising expenditures and the own-price supply elasticity, the elasticity of substitution between inputs, and trade elasticities. The author estimated the optimal amount of generic advertising for the Canadian cattle industry in a liberalized trade environment.

Perrin (1980) examined the potential surplus gains to producers and consumers that could occur if commodity pricing were expanded into component pricing. The author considered soybean prices (which do not reflect oil or fat content) and milk prices (which do not reflect nonfat solids). Perrin showed that component pricing can generate surplus gains for both consumers and producers through improved price signals, which may be too expensive to accommodate using usual arbitrage processes.

Zhao, Anderson, and Wittwer (2003) developed a vertically and horizontally disaggregated EDM of the Australian wine market. The model was horizontally separated into premium and nonpremium markets and included sectors for grape producers, wine makers, wine advertisers, and foreign and domestic consumers. This disaggregation allowed the authors to analyze the surplus effects of promotion, research, and market development activities for each of these levels and markets and to consider the effects of a split-tax market enhancement program on grape producers and wine makers.

Kinnucan (1997) built on the existing literature of interrelated industry advertising in agricultural markets by incorporating input substitution at the processing level. He focused on how processing level input substitutability affects the benefits of demand shifts caused by advertising. Using results from Brester and Schroeder (1995) and Kinnucan, Xiao, and Hsia (1996), he confirmed the direction of bias and estimated returns to advertising. Kinnucan suggested that future models should include processor behavior at the outset to remove these biases.

Market Power

Freebairn, Davis, and Edwards (1982) examined changes in consumer and producer surplus from research in multistage agricultural production systems. They showed that research at any stage benefits all stages equally. Using assumptions that included linear supply and demand curves, a perfectly elastic supply of marketing activities, and competitive pricing, they estimated similar increases in producer surplus at each stage of production. The authors considered cases of perfectly inelastic production inputs as well as perfectly competitive markets on the distribution of gains from research activities. Under each of these assumptions, they treated research gains as parallel shifts of supply curves and developed a general competitive model. They also addressed imperfectly competitive markets under a similar set of assumptions.

Holloway (1991) expanded Wohlgenant's (1989) EDM of eight agricultural sectors and included the potential for imperfect competition in each. He found that these markets generate equilibria similar to those obtained from a perfectly competitive solution. Holloway's EDM addressed the criticism of assuming perfect competition in such models and created a framework for examining long-run producer behavior, incentives for cooperation, and strategies for preventing market entry by incumbents.

Wohlgenant and Piggott (2003) used an EDM to consider the behavior of farm-retail price spreads in the presence of potential food processing market power. Previous research assumed perfectly competitive intermediate markets while allowing input substitution and other factors to determine surplus gains to producers from these programs. They evaluated the effects of research and promotion funded by producers when market intermediaries have market power. The authors tested the influence of market power on the U.S. beef market using parameter estimates that could encompass several market structures. The results showed that supply and demand elasticities and input substitutability remain the most important factors for determining the size of surplus effects generated by research and development, even under imperfect competition. The results from Monte Carlo simulations showed that market power at the intermediate level did not have a statistically significant effect on advertising and promotion effects at nonfarm levels.

Wohlgenant (1999a) used an EDM to evaluate the effects of product heterogeneity on the relationship between retail and farm prices. He found that observed markup behavior in the processing sector may be erroneously attributed to market power if product differentiation exists. The results indicate the importance of including input substitution in such research efforts.

A report by RTI International (2007a) used an EDM to study the effects of alternative marketing arrangements in the U.S. fed cattle and beef sectors. The model illustrated the impact of changing levels of market power on price and quantity equilibria as well as consumer and producer surplus. The study also presented confidence intervals for each of these estimates. A similar analysis was also conducted with respect to the U.S. lamb industry (RTI International, 2007b).

International Trade

Sumner and Wohlgenant (1985) used an EDM to estimate the effects of an increase in federal excise taxes on U.S. tobacco producers' quota lease rates, revenue, and rents. They developed an EDM using log-linear market supply and demand equations. In addition to domestic supply, the authors included an international trade sector. They expanded the Muth (1964) and Gardner (1975) models using derived supply and demand functions and applied a proportional change in the excise tax rate. The model was also unique in that policy responses by the U.S. Department of

Agriculture were treated endogenously, allowing for different outcomes depending on how the federal sector altered tobacco quotas in response to tax increases. The results underscore the importance of accounting for trade linkages. The effects on producers' returns and tobacco prices were significant. On the other hand, the incidence of the tax increase was primarily borne by consumers, even in the case of a fixed tobacco quota.

Alston, Norton, and Pardey (1995) expanded on the approach of Sumner and Wohlgenant (1985) by including data from every country that produces and consumes 28 agricultural commodities. They specified supply and demand functions for each commodity and associated market-clearing conditions. The model included specific institutional constraints across countries. Wohlgenant (1999b) used a similar model to evaluate the effect of trade liberalization on the world sugar market. The EDM included price wedges representing the effects of bounded tariffs on sugar imports. Brester and Wohlgenant (1997) developed an EDM to evaluate the impact of reduced multilateral trade restrictions related to the GATT/Uruguay Round trade negotiations on U.S. beef and cattle prices. In addition, they considered the sensitivity of their predictions to various demand and supply elasticity estimates.

Çakır, Boland, and Wang (2018) investigated the economic impacts of the 2015 highly pathogenic avian influenza (HPAI) outbreak on turkey producers in Minnesota and the United States. Because own-price elasticities of demand for turkey were not available, the authors first estimated a system of demand equations for chicken, turkey, beef, and pork to obtain them. They used the results to create an EDM to model the impacts of HPAI on the Minnesota and U.S. turkey sectors. The cost of the HPAI outbreak on U.S turkey producers was substantial, and most losses were caused by reduced exports. However, the authors found that losses could have been higher had it not been for the implementation of regional bans by trading partners who had previously negotiated free trade agreements with the United States.

Dynamic Responses

An EDM framework can also accommodate dynamic responses in supply and/or demand relations. However, accommodating dynamics in measuring surplus changes is more challenging. One way to incorporate these dynamics is to use more responsive demand and supply elasticities for more distant time periods when calculating changes in consumer and producer surplus (Just, Hueth, and Schmitz, 2004). Lemieux and Wohlgenant (1989) used this approach to estimate the effects of introducing porcine somatotropin in the pork industry by altering elasticities based on time and using standard formulas to estimate surplus effects for the short, intermediate, and long run.

Gotsch and Wohlgenant (2001) evaluated the effects of adopting and producing new cocoa cultivars on consumer and producer surplus. Their analysis incorporated dynamic perennial crop supply responses to changes in cocoa prices. They modeled dynamics by shifting the supply curve for each period based on the effects that early adoption and planting decisions were likely to have on the net present value of producer returns. They calculated price, quantity, and surplus effects for each period and used net present values to derive their conclusions. They also considered both parallel and pivotal supply shifts in response to technological changes.

Weaber and Lusk (2010) evaluated the effects of a genetic improvement program in the cattle industry using discounted net present values of returns over a five-year period. The approach is similar to that of Lemieux and Wohlgenant (1989) but with explicit attention given to discounting future net returns when calculating changes in producer surplus. Pendell et al. (2010) considered the impacts of adopting animal identification and tracing systems on the U.S. meat and livestock industry using a multi-market EDM. They used both short- and long-run elasticity estimates to evaluate changes in market equilibria that result from increased costs of animal identification systems. Further, they considered the long-run impacts on changes in producer and consumer surplus by using discounted present values of annual impacts over a 10-year period.

Precision of EDM Estimates

When using EDMs to estimate changes in prices, quantities, and surplus values, the sensitivity of the results to various plausible supply, demand, and input substitution elasticity estimates is an important issue. An elasticity sensitivity analysis may be all that is needed if a researcher is only concerned about a subset of parameters. However, a high degree of uncertainty regarding parameter values may require the use of a more unified approach such as that developed by Davis and Espinoza (1998) or Zhao et al. (2000a).

Davis and Espinoza (1998) proposed a Bayesian approach for developing confidence intervals for EDM outcomes that generates empirical probability distributions for reduced-form parameters. The procedure, while analogous to a Monte Carlo simulation, is more general in that a researcher's subjective values of parameters can be incorporated into the model, which allows for more informative simulation analyses. Lee, Sumner, and Champetier (2019) used a similar approach in their EDM of the almond and pollination industries.

Zhao and Griffiths (2000) argued that using a normal distribution for elasticity estimates allows for more precision while maintaining the accuracy of those estimates. They compared their probability estimates with those of Davis and Espinoza (1998) using Monte Carlo simulations to produce distributions that fall within acceptable ranges and are more precise. Further, they allowed for the potential of choosing an incorrect parameter estimate conditional on EDM outcomes. They replicated Davis and Espinoza's parameter estimates for price changes and advocated using normal distributions.

Zhao et al. (2000a) used stochastic parameter estimates to examine the sensitivity and robustness of results obtained by Mullen, Alston, and Wohlgenant (1989) regarding technological change in the Australian wool industry. Using a Bayesian approach, the authors showed that parameter estimates can be generated using probability distributions that allow for robustness. Previous parameter estimates were subjective and based on researchers' knowledge or on previously estimated estimates. Further, as models increase in complexity, using subjective parameter estimates for robustness testing becomes difficult in the presence of parameter uncertainty. Using stochastic parameter estimates, the authors showed that Monte Carlo simulations can be used to generate probability distributions of surplus returns from technological innovation. Although the initial estimates were relatively robust, the underlying production groups were sensitive to the stochastic results.

Brester, Marsh, and Atwood (2004) considered the effect of country-of-origin labeling on the U.S. beef, pork, and poultry sectors. They considered the sensitivity of changes in prices and quantities as well as producer and consumer surplus relative to selected elasticity estimates. Following Davis and Espinoza (1998), they conducted Monte Carlo simulations of an EDM by selecting prior distributions for each of the elasticities used in the model. They incorporated diffuse priors with respect to reported demand and supply elasticities and assumed that own-price demand elasticities are always negative, and that own-price supply elasticities are always positive. They noted that a sensitivity analysis of an EDM should not only consider variations of elasticity estimates but also correlations among elasticities because primary and derived supply elasticities are likely correlated within meat sectors. The simulation required sampling from selected distributions, but the existence of correlated elasticities distorts marginal distributions. Rather than sampling from an intractable multivariate distribution, they used a variant of Iman and Conover's (1982) procedure for generating a correlated multivariate sample. They accounted for distortions of marginal distribution samples caused by correlations among elasticities by reordering individually and independently generated marginal samples. Empirical distributions were generated for each endogenous variable and for all estimates of changes in consumer and producer surplus. These empirical distributions were used to develop means, confidence intervals, and *p*-values for price, quantity, and surplus measures.

Summary

EDMs have been widely used to evaluate many policy issues. Their flexibility for studying a variety of economic problems has made them a popular tool since their introduction in the 1950s. Agricultural economists have widely used EDMs for various public policy issues. Since the 1930s, the U.S. Congress has passed agricultural policy legislation every five years or so. Authorized programs often require cost analyses before Congress passes an appropriation bill to fund them. This literature review touched on many of these programs, including advertising and promoting various agricultural products, public investments in research and development, labeling laws, and trade policies. By using EDMs to evaluate these policies, agricultural economists have made pioneering contributions to the development and refinement of these models and resulting policy prescriptions.

While researchers have used EDMs to study policy issues in other industries, the majority of EDM research has explored the agricultural, environmental, and natural resource sectors, which are the target of many legislative policies. Consequently, leading doctoral programs with their historical roots in agricultural economics have taught these concepts and graduates of those programs have developed careers in public universities and government agencies, including the U.S. Department of Agriculture. The formulation of EDMs has evolved as new data have become available and researchers have built upon previous research in their use of these models. For example, as agricultural supply chains have trended toward oligopolistic structures and/or contractual arrangements, EDMs have been used to study departures from perfectly competitive market structures.

What Is a Farm Bill?

Modern Farm Bills include nutrition programs such as food stamps, assistance for farmers, and other programs. Farm bills have been written approximately every five years since the mid-1960s, but Federal aid programs for farmers have existed since the 1930s. Farm bills include mandatory spending programs, which generally operate as entitlements. The costs of these programs are based on multi-year or baseline budget estimates. A farm bill may also authorize, but not fund, discretionary spending programs making them subject to annual appropriations.

Legislated farm policies are wide-ranging and address many specific issues. For example, the 2018 Farm Bill contains 12 titles: Title I, Commodity Programs, provides price support for some (but not all) crops, including wheat, corn, soybeans, peanuts, and rice. It includes disaster programs to help livestock and tree fruit producers manage production losses due to natural disasters. Other support includes margin insurance for dairy, marketing quotas, minimum price guarantees, and sugar import restrictions.

Title II, Conservation, encourages: (1) environmental stewardship and improved management practices; (2) working lands programs, including the Environmental Quality Incentives Program and the Conservation Stewardship Program; (3) land retirement programs, including the Conservation Reserve Program; and (4) other aid, such as the

Agricultural Conservation Easement Program and the Regional Conservation Partnership Program.

Title III, Trade, supports U.S. agricultural export programs and international food assistance, including the Market Access Program and the primary U.S. international food aid program, Food for Peace. This title also includes program changes related to World Trade Organization obligations.

Title IV, Nutrition, provides nutrition assistance for low-income households through programs such as the Supplemental Nutrition Assistance Program (formerly known as food stamps), the Emergency Food Assistance Program, and food distribution in schools.

Title V, Credit, offers direct government loans to farmers and ranchers, guarantees loans provided by commercial lenders, and sets eligibility rules and policies.

Title VI, Rural Development, supports rural business and community development, provides for planning and feasibility assessments, and coordinates other local, state, and Federal programs including grants and loans for infrastructure, economic, broadband, and telecommunications development.

Title VII, Research, Extension, and Related Matters, offers a wide range of agricultural research and extension programs that expand knowledge regarding agriculture and food and helps farmers and ranchers become more efficient, innovative, and productive.

Title VIII, Forestry, supports forestry management programs operated by USDA's Forest Service.

Title IX, Energy, encourages the development of farm and community renewable energy systems through grants, loan guarantees, and feedstock procurement initiatives. It includes provisions that address the production, marketing, and processing of biofuels and biofuel feedstock and research, education, and demonstration programs.

Title X, Horticulture, supports certified organic and specialty crop (fruits, vegetables, tree nuts, and floriculture and ornamental products) production through a range of initiatives—including market promotion, plant pest and disease prevention, and public research.

Title XI, Crop Insurance, comprises the permanently authorized federal crop insurance program.

Title XII, Miscellaneous, addresses other issues, including livestock and poultry production as well as limited-resource and socially disadvantaged farmers.

The scope and scale of these programs have required, and provided, wide-ranging research opportunities for economists to use EDMs to study the impacts of each.

»Chapter Three

MATHEMATICAL TOOLS FOR EVALUATING MARKET PERTURBATIONS

An understanding of linear algebra and the calculus of differentials is essential for developing and using equilibrium displacement models (EDMs). Appendix 3A presents a brief review of linear algebra and differentiation. It is also important to note that all economic modeling involves approximations to underlying, unknown functional forms. EDMs consist of linear approximations of unknown demand and supply functions. The accuracy of these approximations depends upon the size of the perturbation considered and the degree of nonlinearity of the functional forms being approximated. EDMs and comparative statics models provide equivalent outcomes for linear approximations of nonlinear functions. We illustrate this equivalency using a numerical example from a total differential approach and an EDM approach.

Evaluating Perturbations to a Single Linear Function

We first use a linear equation to trace the exact change in a dependent variable caused by an exogenous shock to an independent variable. We then illustrate the equivalency of a total differential approach and an EDM approach using a numerical example.

Consider the following linear equation, which represents a simplified demand function:

(3.1) $$q^D = a - \alpha p^D,$$

where q^D represents the quantity demanded of a good, p^D represents the demand price of a good, a is an intercept, and α is the slope of the linear function. Assume that the function is parameterized with $a = 10$, $\alpha = 1$, $p_0^D = 5$, and $q_0^D = 5$, where subscripts represent an initial price and quantity.

Numerical Evaluation of a Perturbation to a Linear Equation

The goal is to first find an exact numerical value for the dependent variable given an exogenous shock to the independent variable in (3.1). Assume that price changes from p_0^D to p_1^D by 0.5 units, such that $dp^D = 0.5$. Consequently, the new ending value, q_*^D, is given by

(3.2) $$q_*^D = 10 - 1(p_0^D + dp^D) = 10 - 1(5.5) = 4.5.$$

A Total Differential Approach for Evaluating a Perturbation of a Linear Equation

Total differentials can be used to evaluate a change in p^D on the dependent variable q^D. To do so, we first totally differentiate (3.1) to obtain

(3.3) $$dq^D = \frac{dq^D}{dp^D} dp^D = -\alpha dp^D.$$

For the values $a = 10$, $\alpha = 1$, $p_0^D = 5$, and $q_0^D = 5$, we find that $\frac{dq^D}{dp^D} = -1$. If we again let $dp^D = 0.5$, then

(3.4) $$dq^D = -1dp^D = -0.5 \text{ and}$$

(3.5) $$q_*^D = q_0^D + dq^D = 5.0 - 0.5 = 4.5,$$

which is the exact result obtained using the numerical approach shown in (3.2).

A Proportional Elasticity Approach to Evaluating a Perturbation of a Linear Equation

Proportional elasticity equations provide the basis for EDMs. This approach to quantifying the effects of a perturbation yields the same answer as the previously illustrated numerical and total differential approaches because (3.1) is linear. A proportional elasticity equation is developed by, again, totally differentiating (3.1) to yield

(3.6) $$dq^D = \frac{dq^D}{dp^D} dp^D = -\alpha dp^D.$$

Next, multiply both sides of (3.6) by $\frac{1}{q^D}$, which yields

(3.7) $$\frac{dq^D}{q^D} = -\alpha \frac{1}{q^D} dp^D.$$

Now, multiply the right-hand side of (3.7) by $\frac{p^D}{p^D}$ to obtain

(3.8) $$\frac{dq^D}{q^D} = -\alpha \frac{p^D}{p^D} \frac{dp^D}{q^D} = -\alpha \frac{p^D}{q^D} \frac{dp^D}{p^D}.$$

Note that $-\alpha \frac{p^D}{q^D}$ represents the own-price elasticity of demand, η^D, for q^D. By allowing $E(\cdot)$ to represent proportional changes, (3.8) can be rewritten as

$$E(q^D) = \eta^D E(p^D). \tag{3.9}$$

Given our assumed initial value of p_0^D, a change in p^D, or dp^D, of 0.5 units results in $E(p^D) = \frac{0.5}{5} = 0.10$. Further, if $\alpha = 1$, $p_0^D = 5$, and $q_0^D = 5$, then $\eta^D = -1.0$. The proportional change in q^D, $E(q^D)$, is found by multiplying η^D by $E(p^D)$ such that

$$E(q^D) = \eta^D E(p^D) = (-1)(0.10) = -0.1. \tag{3.10}$$

In addition, proportional changes imply that the new equilibrium is found by multiplying the original equilibrium, q_0^D, by 1 plus the proportional change in q^D, which yields

$$q_*^D = (q_0^D)(1 + E(q^D)) = (5)(1 + (-0.1)) = (5)(0.9) = 4.5. \tag{3.11}$$

The proportional elasticity approach yields the same new equilibrium quantity, 4.5, as both the numerical approach and the total differential comparative static approach. This result occurs because the initial equation of interest, (3.1), is linear.

Evaluating Perturbations to a Nonlinear Equation

Consider a change in an endogenous variable caused by a change in an exogenous variable for a *nonlinear* equation. Because of the nonlinearity, the exact numerical outcome of an exogenous shock is not equivalent to either a total differential approximation or a proportional elasticity approach. However, the latter two are equivalent in their approximation of the actual outcome. Consider the following nonlinear equation, which represents a simplified demand function:

$$q^D = \alpha(p^D)^{-\beta}, \tag{3.12}$$

where q^D represents the quantity demanded of a good, p^D represents the price of a good, and α and β are parameters. For this example, allow (3.12) to be parameterized with $\alpha = 18$, $\beta = 1$, $p_0^D = 3$, and $q_0^D = 6$.

A Numerical Evaluation of a Perturbation to a Nonlinear Equation

Assume that the value of p_0^D is exogenously altered such that the change in p^D is given by $dp^D = 0.3$. The effect of this change on the dependent variable is calculated as

(3.13) $$q_*^D = a(p_0^D + dp^D)^{-\beta} = 18(3.0 + 0.3)^{-1}$$
$$= 18(3.3)^{-1} = 5.4545.$$

A Total Differential Evaluation of a Perturbation to a Nonlinear Equation

To obtain a comparative statics solution to the above problem, the first step is to totally differentiate (3.12), which yields

(3.14) $$dq^D = -\beta a(p^D)^{-\beta-1}dp^D = -\beta a(p^D)^{-\beta}\frac{1}{p^D}dp^D.$$

Substituting q^D from (3.12) into (3.14) yields

(3.15) $$dq^D = -\beta\frac{q^D}{p^D}dp^D.$$

Once again, allow $\alpha = 18$, $\beta = 1$, $p_0^D = 3$, $q_0^D = 6$, and $dp^D = 0.3$. The total differential approach results in a linearly approximated value to the change in p^D:

(3.16) $$q_*^D = q_0^D + dq^D = 6 + (-1\left(\frac{6}{3}\right)(0.3))$$
$$= 6 + (-2)(0.3) = 6 - 0.6 = 5.40.$$

The value of 5.40 is not identical to the true value of 5.4545 obtained in (3.13).

A Proportional Elasticity Evaluation of a Perturbation to a Nonlinear Equation

EDMs use proportional elasticity equations to evaluate the change in a dependent variable caused by a change in an independent variable. A proportional elasticity approach yields an identical answer to that obtained from a total differential approach for nonlinear equations. However, neither provides the exact answer to the true numerical solution. To illustrate, totally differentiate (3.12) and convert to percentage changes and elasticities to obtain

(3.17) $$E(q^D) = \eta^D E(p^D).$$

If we once again allow the change in p^D to be $dp = 0.3$, then $E(p^D) = \frac{0.3}{3} = 0.10$. At the point $p_0^D = 3$, $q_0^D = 6$, the slope of a linear approximation at the initial equilibrium point is calculated by substituting $p_0^D = 3$, $q_0^D = 6$, and $\beta = 1$ into (3.15) to yield

(3.18) $$\frac{dq^D}{dp^D} = -1\left(\frac{6}{3}\right) = -2.0.$$

Thus, the elasticity of demand at the initial equilibrium point is

$$(3.19) \quad \eta^D = \frac{dq^D}{dp^D}\frac{p^D}{q^D} = -2.0\left(\frac{3}{6}\right) = -1.0.$$

Substituting the values for η^D and $E(p^D)$ into (3.17) yields

$$(3.20) \quad E(q^D) = (-1.0)(0.1) = -0.1.$$

Therefore, the new equilibrium quantity given the assumed exogenous change in price is calculated as

$$(3.21) \quad q_*^D = q_0^D(1 + E(q^D)) = 6(1 - 0.1) = 5.40.$$

This answer is identical to that obtained using the comparative statics approach from (3.16). However, it is not identical to the true value, 5.4545, as calculated in (3.13).

Summary

EDMs are linear approximations of unknown, and likely nonlinear, demand and supply functions. The accuracy of any model that uses linear approximations to nonlinear functions for estimating changes in equilibria depends upon the size of perturbation considered and the degree of the true nonlinearity of the functional form being approximated. If the underlying functions are actually linear, then the total differential and proportional elasticity approaches provide exact solutions of the change in an endogenous variable caused by a change in an exogenous variable.

However, if the underlying functional form being approximated is nonlinear, then neither a total differential nor a proportional elasticity approach will yield an exact solution of changes in an endogenous variable resulting from an exogenous shock. Nonetheless, both approaches yield identical, although biased, results. The extent of the bias depends upon the size of the perturbation being considered and the degree of nonlinearity of the underlying function. The next chapter develops an EDM using proportional elasticity equations. Readers who require a refresher in linear algebra and differential calculus are encouraged to review Appendix 3A before considering Chapter 4.

From: Student@UEconomics.edu
To: Professor Watson
Date: Thursday, 14 Nov 2021 at 3:47 p.m.
Subject: Nonlinearities

You note that supply and demand curves are likely nonlinear, and that EDMs are linear approximations to these curves. Hence, they should only be used to approximate changes in equilibria for relatively small exogenous shocks. How do we know that supply and demand curves are nonlinear and to what extent do we think their curvature is?

Thanks!
Mark

From: Professor Watson
To: Student@UEconomics.edu
Date: Friday, 15 Nov 2021 at 9:17 a.m.
Re: Nonlinearities

Dear Class,

Great question and it is easier for me to understand this after a lifetime of being an agricultural economist. We don't live in a linear world. Part of my dissertation involved conducting research with animal scientists to evaluate economically optimal harvesting weights for cattle because the economically optimal weight is always less than the biologically optimal weight. Many of these supply functions are exponential in nature, which suggests an asymptote. This is caused by underlying physical constraints and resulting production functions. Benjamin Gompertz, a nineteenth-century actuary living in England, wrote a well-known paper on mortality. The Gompertz functional form has been widely used in physical sciences for modeling growth. And we have not talked much about von Liebeg production functions, which generate fixed input proportions, and the constraints that occur when an input is limited within a production process.

Consider the following: The world record for the 1600 meter race is much more than 4 times the world record for the 400 meter dash. We cannot grow at the same linear rate forever (although I sometimes question that when it comes to my personal weight gain!). There is a limit to our height and weight as humans. We live in a world of biological and physical limitations. Why would we think that we live in a linear economic world given the biological, physical, and mechanical constraints that underlie all economic activity?

It is likely that—for the reasons described above—most agricultural supply curves resemble some form of exponential function. It could be that the shape of demand curves is closer to linearity than supply curves because biology doesn't provide a constraint on human desires. Think about the components of a demand curve such as income, price of substitutes, expectations of future prices, own price, and consumer tastes and preferences. Four of the five components are likely to be step functions because they are expressed in "money" concepts and probably lack infinite divisibility for an individual consumer, although they may be almost infinitely divisible in an entire market. Perhaps this generates less curvature in individual demand functions relative to supply functions. Tastes and preferences are more difficult to quantify, but economic research using experimental auctions shows that demand curves are nonlinear, as would be expected. They may not, however, be of an exponential form within a given range of data. The bottom line is that we should not expect supply and demand curves to be linear, although demand curves are more likely to be closer to linearity than supply curves. Nonetheless, we often teach economics using linear depictions of these functions, but this is only for the sake of simplicity and illustrative clarity.

All the best,
Dr. Watson

»Chapter Three, Appendix A

REVIEW OF LINEAR ALGEBRA AND DIFFERENTIAL CALCULUS

The fundamentals of matrix algebra and differentials are essential for understanding, developing, and using EDMs. This appendix provides the basics of linear algebra and differential calculus and is designed for those who have a limited understanding of these powerful mathematical tools. Total differentiation is used to develop equations in proportional elasticity form and linear algebra is used to simultaneously find solutions to multiple endogenous variables caused by changes in one or more exogenous variables in these equations.

Consider a system of two linear equations:

(3A.1) $$a_{11}y_1 + a_{12}y_2 = x_1$$

$$a_{21}y_1 + a_{22}y_2 = x_2,$$

where a_{ij} are parameters, y_i are endogenous variables, and x_i are exogenous variables.

This system can be written in matrix notation as

(3A.2) $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and can be written more compactly as

(3A.3) $$\boldsymbol{Ay} = \boldsymbol{x},$$

where $\boldsymbol{A}$ is a 2×2 matrix, $\boldsymbol{y}$ is a 2×1 vector, and $\boldsymbol{x}$ is a 2×1 vector.

Rules of Matrix Operations

The following sections present the basic rules of matrix operations.

Addition and Subtraction

The dimension of a matrix is determined by its number of rows and columns. The addition or subtraction of two matrices requires that both have the same dimensions. To add two matrices, each element of each matrix is added to its corresponding element. For example, consider the following 2×2 matrices:

(3A.4) $$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } \boldsymbol{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

Adding the two matrices results in:

(3A.5) $$\boldsymbol{A} + \boldsymbol{B} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}.$$

For a numerical example, consider two matrices such as

(3A.6) $$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The sum of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ is

(3A.7) $$\boldsymbol{A} + \boldsymbol{B} = \begin{bmatrix} 1+1 & 2+0 \\ 3+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}.$$

To subtract $\boldsymbol{B}$ from $\boldsymbol{A}$, the general notation is

(3A.8) $$\boldsymbol{A} - \boldsymbol{B} = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}.$$

Numerically, the result is given by

(3A.9) $$\boldsymbol{A} - \boldsymbol{B} = \begin{bmatrix} 1-1 & 2-0 \\ 3-0 & 4-1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 3 \end{bmatrix}.$$

Scalar Multiplication

A matrix of any dimension can be multiplied by a scalar. The process involves multiplying each element of a matrix by the scalar of interest. In general, multiplying matrix $\boldsymbol{A}$ by a scalar k yields

(3A.10) $$k\boldsymbol{A} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}.$$

Numerically, if $k = 3$ and we use the $\boldsymbol{A}$ matrix noted above, then

(3A.11) $$k\boldsymbol{A} = 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 & 3 \cdot 2 \\ 3 \cdot 3 & 3 \cdot 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}.$$

Matrix Multiplication

Two matrices can be multiplied only if the number of columns in the first matrix is equal to the number of rows in the second matrix. The row dimension of the product is equal to the number of rows in the first matrix and the column dimension of the product is equal to the number of columns of the second matrix. In general,

(3A.12) $$\boldsymbol{A}_{ixp}\boldsymbol{B}_{pxj} = \boldsymbol{C}_{ixj}.$$

Each element of $\boldsymbol{C}$ is found by multiplying the i^{th} row of $\boldsymbol{A}$ by the j^{th} column of $\boldsymbol{B}$ such that

(3A.13) $$\boldsymbol{C} = \begin{bmatrix} (a_{11} \cdot b_{11}) + (a_{12} \cdot b_{21}) & (a_{11} \cdot b_{12}) + (a_{12} \cdot b_{22}) \\ (a_{21} \cdot b_{11}) + (a_{22} \cdot b_{21}) & (a_{21} \cdot b_{12}) + (a_{22} \cdot b_{22}) \end{bmatrix}.$$

Using the above values for $\boldsymbol{A}$ and $\boldsymbol{B}$, the product of the two matrices is

(3A.14) $$\boldsymbol{C} = \begin{bmatrix} (1 \cdot 1) + (2 \cdot 0) & (1 \cdot 0) + (2 \cdot 1) \\ (3 \cdot 1) + (4 \cdot 0) & (3 \cdot 0) + (4 \cdot 1) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Matrix Transposition

The transpose of a matrix results from taking the i^{th} row of a matrix and making it the i^{th} column in the transposed matrix. Therefore, the transposition of matrix $\boldsymbol{A}$ in (3A.4) is given by

(3A.15) $$\boldsymbol{A}' = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}.$$

Numerically, the transpose of matrix $\boldsymbol{A}$ in (3A.6) is

(3A.16) $$\boldsymbol{A}' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}.$$

Properties of Matrix Transposition

Property 1: $(\boldsymbol{A}')' = \boldsymbol{A}$

Property 2: $(\boldsymbol{A} + \boldsymbol{B})' = \boldsymbol{A}' + \boldsymbol{B}'$

Property 3: $(\boldsymbol{AB})' = \boldsymbol{B}'\boldsymbol{A}'$

Properties of Matrix Operations

Property 1: $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}$ and $\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} = (\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C} = \boldsymbol{C} + \boldsymbol{B} + \boldsymbol{A}$

Property 2: $\boldsymbol{ABC} = (\boldsymbol{AB})\boldsymbol{C} = \boldsymbol{A}(\boldsymbol{BC})$

Property 3: $\boldsymbol{A}(\boldsymbol{B} + \boldsymbol{C}) = \boldsymbol{AB} + \boldsymbol{AC}$ and $(\boldsymbol{A} + \boldsymbol{B})\boldsymbol{C} = \boldsymbol{AC} + \boldsymbol{BC}$

$$(\boldsymbol{A} + \boldsymbol{B})(\boldsymbol{C} + \boldsymbol{D}) = \boldsymbol{AC} + \boldsymbol{AD} + \boldsymbol{BC} + \boldsymbol{BD}$$

$$\boldsymbol{ABC} + \boldsymbol{AFC} = \boldsymbol{A}(\boldsymbol{B} + \boldsymbol{F})\boldsymbol{C}$$

Property 4: An identity matrix, $\boldsymbol{I}_n$, must be square ($n \times n$) and have only entries of 1 on the diagonal, and 0 for all off-diagonal entries. Multiplying any matrix by an appropriately dimensioned identity matrix leaves the original matrix unchanged:
$\boldsymbol{I}_m \boldsymbol{A}_{mxn} = \boldsymbol{A}_{mxn} \boldsymbol{I}_n = \boldsymbol{A}_{mxn}$.

Special Matrices

Idempotent matrix: A square matrix $\boldsymbol{A}$ is an idempotent matrix if $\boldsymbol{A}\,\boldsymbol{A} = \boldsymbol{A}$.
Symmetric matrix: A square matrix $\boldsymbol{A}$ is symmetric if $\boldsymbol{A}' = \boldsymbol{A}$.
Diagonal matrix: A diagonal matrix is a square matrix that has only 0s as entries in the off-diagonal elements.
Inverse matrix: If an inverse matrix $\boldsymbol{A}^{-1}$ exists for a square matrix $\boldsymbol{A}$, then $\boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{A}\,\boldsymbol{A}^{-1} = \boldsymbol{I}$.

Properties of Inverse Matrices

Property 1: Only square matrices can have inverses.
Property 2: Matrices that have inverses are termed nonsingular matrices.
Property 3: Matrices that do not have inverses are termed singular matrices.
Property 4: If an inverse exists, it is unique.
Property 5: The inverse matrix of $\boldsymbol{A}$ can only exist if all the rows of $\boldsymbol{A}$ are linearly independent and all the columns of $\boldsymbol{A}$ are linearly independent, which makes $\boldsymbol{A}$ of full rank.
Property 6: If $\boldsymbol{AB} = \boldsymbol{C}$ and $\boldsymbol{A}^{-1}$ exists, then $\boldsymbol{A}^{-1}\boldsymbol{AB} = \boldsymbol{A}^{-1}\boldsymbol{C} \Leftrightarrow \boldsymbol{B} = \boldsymbol{A}^{-1}\boldsymbol{C}$.

Determinants

If a square matrix $\boldsymbol{A}$ is of full rank, its inverse $\boldsymbol{A}^{-1}$ exists. Determinants represent one of a variety of tests that can be used to determine if a square matrix is of full rank. In general terms, the determinant $|\boldsymbol{A}|$ of the matrix

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{3A.17}$$

is

(3A.18) $$|\boldsymbol{A}| = (a_{11}a_{22}) - (a_{21}a_{12}).$$

If the determinant of $\boldsymbol{A}$ is not 0, then the matrix is of full rank and nonsingular. Numerically, let

(3A.19) $$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Then, the determinant of $\boldsymbol{A}$ is

(3A.20) $$|\boldsymbol{A}| = (1 * 4) - (3 * 2) = 4 - 6 = -2.$$

Because the determinant of $\boldsymbol{A}$ is nonzero, matrix $\boldsymbol{A}$ is of full rank, nonsingular, and its inverse exists.

Laplace Expansion for Calculating Determinants

The Laplace expansion is one of several approaches that can be used to calculate the determinants of higher-dimension matrices. The procedure requires that the concepts of minors and cofactors be defined. Let

(3A.21) $$\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Minor: The ij^{th} minor of $\boldsymbol{A}$ is $M_{ij} \equiv$ the determinant of the matrix remaining after the i^{th} row and the j^{th} column have been deleted from matrix $\boldsymbol{A}$. Numerically, the M_{11} minor is

(3A.22) $$M_{11} = \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = -3.$$

Cofactor: The ij^{th} cofactor of $\boldsymbol{A}$ is its "signed minor" defined as $C_{ij} \equiv (-1)^{i+j}M_{ij}$. Numerically, the cofactor C_{11} is

(3A.23) $$C_{11} = (-1)^{1+1}\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = (-1)^2(-3) = -3.$$

The Laplace expansion can be used to find the determinant of $\boldsymbol{A}$ through either a row i or column j expansion such that

(3A.24) $$|\boldsymbol{A}| = \sum_{i \text{ or } j} a_{ij}C_{ij} = \sum_{i \text{ or } j} a_{ij}(-1)^{i+j}M_{ij}.$$

Using the first row to expand, the determinant of $\boldsymbol{A}$ is calculated as

(3A.25) $$|\boldsymbol{A}| = (2)(1)\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} + (1)(-1)\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + (3)(1)\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= (2)(-3) + (-1)(-6) + (3)(-3) = -9.$$

Properties of Determinants

Property 1: Determinants are only defined for square matrices.

Property 2: $|\boldsymbol{A}| = |\boldsymbol{A}'|$.

Property 3: If any two rows or any two columns of a square matrix are interchanged, the magnitude of the determinant is not altered, but its sign is reversed.

Property 4: The multiplication of any row or column by a scalar also multiplies the determinant by the same scalar.

Property 5: If a scalar is multiplied by the i^{th} row (or the j^{th} column) and the result is added to the j^{th} row (or the i^{th} column), the determinate is not altered.

Property 6: If at least one row (or column) is a linear combination of the remaining rows (or columns), then the determinant is 0.

Property 7: Any matrix that has a determinant of 0 is singular.

Solving Systems of Linear Equations

Linear algebra is used to solve systems of linear equations such as those represented by (3A.1). Several methods can be used to accomplish this goal. One common method for finding a solution for a single endogenous variable is by using Cramer's Rule. A second method, inverses, is used to find solutions for multiple endogenous variables.

Cramer's Rule

To solve for x_i in (3A.2), Cramer's Rule states that

(3A.26) $$y_i = \frac{|\boldsymbol{A}_i|}{|\boldsymbol{A}|},$$

where $|\boldsymbol{A}_i|$ is the determinant of the matrix formed by replacing the i^{th} column of $\boldsymbol{A}$ with the vector $\boldsymbol{x}$. For equation (3A.2), the value of y_1 is calculated as

(3A.27) $$y_1 = \frac{|\boldsymbol{A}_1|}{|\boldsymbol{A}|} = \frac{\begin{vmatrix} x_1 & a_{12} \\ x_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{(x_1 a_{22}) - (x_2 a_{12})}{(a_{11}\, a_{22}) - (a_{21} a_{12})}.$$

Using the values for $\boldsymbol{A}$ presented in (3A.6), the solution for y_1 is found as

$$(3A.28)\quad y_1 = \frac{\begin{vmatrix} x_1 & 2 \\ x_2 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{(x_1)(4)-(x_2)(2)}{(1)(4)-(3)(2)} = \frac{(4x_1)-(2x_2)}{-2} = -2x_1 + x_2.$$

Likewise, for y_2,

$$(3A.29)\quad y_2 = \frac{\begin{vmatrix} 1 & x_1 \\ 3 & x_2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{(1)(x_2)-(3)(x_1)}{(1)(4)-(3)(2)} = \frac{(x_2)-(3x_1)}{-2} = \frac{3}{2}x_1 - \frac{1}{2}x_2.$$

Inverse Matrices

Inverse matrices can be used to solve systems of linear equations for multiple variables simultaneously. Although several methods exist for deriving an inverse matrix, consider the inverse of matrix $\boldsymbol{A}$ using a cofactor approach in which

$$(3A.30)\quad \boldsymbol{C} = [C_{ij}],$$

where C_{ij} is the cofactor of the ij^{th} element of $\boldsymbol{A}$. The ij^{th} element of $\boldsymbol{C}$ is the cofactor of the ij^{th} element of $\boldsymbol{A}$. Therefore,

$$(3A.31)\quad \boldsymbol{C}' = [C_{ji}] \Rightarrow C'_{ij} = C_{ji}.$$

If $|\boldsymbol{A}| \neq 0$, then

$$(3A.32)\quad \boldsymbol{A}^{-1} = \frac{1}{|A|}\boldsymbol{C}'.$$

Numerically, let $\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow \boldsymbol{C} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix} \Rightarrow \boldsymbol{C}' = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$, so that

$$(3A.33)\quad \boldsymbol{A}^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}.$$

To solve for the y_i's in (3A.3), premultiply the left- and right-hand sides by $\boldsymbol{A}^{-1}$:

$$(3A.34)\quad \boldsymbol{A}^{-1}\boldsymbol{A}\boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{x}$$

which yields

$$(3A.35)\quad \boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{x}.$$

Numerically,

(3A.36) $\boldsymbol{y} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$

(3A.37) $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$

giving the same solutions as (3A.28) and (3A.29):

(3A.38) $y_1 = -2x_1 + x_2,$

and

(3A.39) $y_2 = \frac{3}{2}x_1 - \frac{1}{2}x_2.$

Nonlinear Functions

Consider the expression

(3A.40) $y = f(x).$

We often want to examine the effect on *y* for a given change (Δ) in *x*, such that

(3A.41) $x^* = x_0 + \Delta x.$

The change in *y* is given by

(3A.42) $y^* = f(x^*) = f(x_0 + \Delta x) \Rightarrow \Delta y = y^* - y^0$

and the average rate of change is

(3A.43) $\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x)-f(x_o)}{\Delta x}.$

Derivatives

The derivative of a function *y* is defined as the limit of $\frac{\Delta y}{\Delta x}$ as $\Delta x \to 0$. We denote this relationship as

(3A.44) $\frac{dy}{dx} \equiv y' \equiv f'(x) \equiv f_x \equiv \underset{\Delta x \to 0}{LIM} \frac{\Delta y}{\Delta x} = \underset{\Delta x \to 0}{LIM} \left(\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}\right).$

For example, if (3A.40) has the following explicit functional form, (3A.45) $y = f(x) = 3 + x^2$ for x≥0,

Then the derivative of this function is

(3A.46) $y' = \frac{dy}{dx} = 2x.$

If (3A.46) is evaluated at a point, say $x = 3$, then the value of the derivative is 6. Figure 3A.1 presents a graph of (3A.45) as well as its tangent line at $x = 3$. The slope of the tangent line to (3A.45) at $x = 3$ is 6. Note that for a small area around $x = 3$, the slope calculated by (3A.46) is approximately equal to the slope of (3A.45).

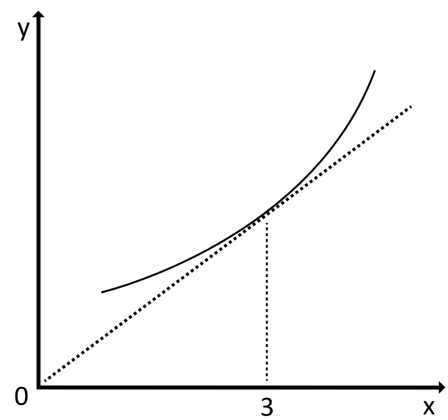

Figure 3A.1. The Derivative of a Nonlinear Function

Rules of Differentiation

Constant Functions

If $y = k$, then $\frac{dy}{dx} = 0$.

Power Functions

If $y = ax^n$, then $\frac{dy}{dx} = nax^{n-1}$.

Multiple Functions of the Same Variable

Sums and Differences of Functions

Consider

$$y = ax^3 + bx^2 + cx + d.$$

The first derivative is given by

$$y' = 3ax^2 + 2bx + c,$$

the second derivative is

$$y'' = 6ax + 2b,$$

and the third derivative is

$$y''' = 6a.$$

For sums or differences of functions, the derivative of the sum is equal to the sum of its derivatives.

Products of Functions

If $y = f(x)g(t)$, then

$$y' = f_x(x)g(x) + f(x)g_x(x).$$

Quotients of Functions

If $y = \frac{f(x)}{g(x)}$, then

$$y' = \frac{f_x(x)g(x) - f(x)g_x(x)}{\left(g(x)\right)^2}.$$

Imbedded Functions

If $y = f(v)$ and $v = g(x)$ such that $y = f(v(x))$, then

$$\frac{dy}{dx} = \frac{dy}{dv}\frac{dv}{dx}.$$

Partial Differentiation

Consider the following expression, which maps two variables, x and z, into a single variable, y:

(3A.47) $$y = f(x, z).$$

Suppose one wished to examine how y is altered by a change in x while the variable z is held constant. The procedure is to partially differentiate (3A.47) while considering variable z a constant, such that

(3A.48) $$\frac{\partial y}{\partial x} \equiv y_x \equiv f_x.$$

Similarly, the partial derivative of y with respect to z is obtained by treating variable x as a constant:

(3A.49) $$\frac{\partial y}{\partial z} \equiv y_z \equiv f_z.$$

For example, consider the following function

$$y = Ax^{\alpha}z^{\beta}. \tag{3A.50}$$

Then,

$$\frac{\partial y}{\partial x} = f_x = \alpha Ax^{\alpha-1}z^{\beta} \tag{3A.51}$$

and

$$\frac{\partial y}{\partial z} = f_z = \beta Ax^{\alpha}z^{\beta-1}. \tag{3A.52}$$

Jacobian Determinants

Consider a system of two equations that may or may not be linear,

$$\begin{aligned} y_1 &= y_1(x_1, x_2, \alpha) \\ y_2 &= y_2(x_1, x_2, \alpha), \end{aligned} \tag{3A.53}$$

and we need to determine whether unique solutions to these simultaneous equations exist so that we can write

$$\begin{aligned} x_1^* &= x_1^*(\alpha) \\ x_2^* &= x_2^*(\alpha). \end{aligned} \tag{3A.54}$$

Jacobian determinants allow both linear and nonlinear independence to be identified in a set of equations. A Jacobian is defined as the matrix of first partial derivatives, such that

$$\boldsymbol{J} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{bmatrix}. \tag{3A.55}$$

If the determinant $|\boldsymbol{J}| = 0$, then the equations are neither linearly nor nonlinearly independent and unique solutions do not exist. For example, consider the following set of equations, where the second is simply the square of the first:

$$\begin{aligned} y_1 &= 2x_1 + 3x_2 \\ y_2 &= 4x_1^2 + 12x_1x_2 + 9x_2^2. \end{aligned} \tag{3A.56}$$

Then,

(3A.57) $$|\boldsymbol{J}| = \left|\begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{bmatrix}\right| = \begin{vmatrix} 2 & 3 \\ 8x_1 + 12x_2 & 12x_1 + 18x_2 \end{vmatrix}$$

$$= 24x_1 + 36x_2 - 24x_1 - 36x_2 = 0.$$

Given that $|\boldsymbol{J}| = 0$, there is no unique solution to the system of equations.

Differentials of Functions

Consider a function of the form

(3A.58) $$y = h(y, x).$$

We are unable to directly evaluate $\frac{\partial y}{\partial x}$ because the variable y appears on both sides of the equation. However, we can develop an expression for $\frac{\partial y}{\partial x}$ using the total differential of (3A.58).

Total Differentials for a Single Variable Function

Consider the following single variable function:

(3A.59) $$y = f(x).$$

The goal is to evaluate a change in y, Δy, given a change in x, Δx. If we multiply Δy by $\frac{\Delta x}{\Delta x}$, we obtain

(3A.60) $$\Delta y = \frac{\Delta y}{\Delta x}\Delta x.$$

The total differential is found by taking the limit of (3A.60) as $\Delta x \to 0$. This results in the differential

(3A.61) $$dy = \frac{dy}{dx}dx = f'(x)dx,$$

which can be viewed as a linear approximation to the original function presented in (3A.59) where the linear approximation occurs along a line that is tangent to the function at the point of interest. It can be shown that this approximation is relatively accurate for changes in sufficiently small areas around the point being evaluated. For example, consider the nonlinear function

(3A.62) $y = x^2.$

The total differential of (3A.62) is

(3A.63) $dy = 2x\, dx.$

Consequently, for a small neighborhood around some initial point x_0, the change in y for a sufficiently small change in x is approximately twice the initial level of x multiplied by the change in x. To illustrate using (3A.62), assume the initial value being considered is $x = 3$, and we want to evaluate the change in y given that x changes from 3 to 3.1. The actual change in y is given by

(3A.64) $\Delta y = (3.1)^2 - (3)^2 = 9.61 - 9 = 0.61.$

Using the total differential noted in (3A.63), the approximate change in y caused by a change in x from 3 to 3.1 is

(3A.65) $dy = (2)(3)(0.1) = 0.60.$

Consequently, for this small change in x, the differential approach yields a change in y that is relatively close to the actual change. It can be shown that the rate of change of the differential converges exactly to the rate of change of the original function as $\Delta x \to 0$.

Total Differentials for Multiple Variable Functions

Consider a function that consists of more than one variable such as

(3A.66) $y = f(x_1, x_2, x_3).$

The total differential of (3A.66) is

(3A.67) $dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 + \frac{\partial y}{\partial x_3} dx_3.$

Thus, the total differential is the sum of the partial derivatives with respect to each right-hand-side variable multiplied by the differential of the variable.

Gradient Vectors

The gradient vector of any function is defined as a vector that contains the first partial derivatives of the function with respect to all endogenous variables. For example, the gradient vector for equation (3A.66) is given by

(3A.68) $$\boldsymbol{y_x} = \begin{bmatrix} \frac{\partial y}{\partial x_1} \\ \frac{\partial y}{\partial x_2} \\ \frac{\partial y}{\partial x_3} \end{bmatrix}.$$

Rules of Differentials

Consider the differentiable functions y, u, and v. Further, define k to be a scalar constant.

Rule 1: If $y = k$, then $dy = \frac{dy}{dk}$.

Rule 2: If $y = cu^n$, then $dy = (ncu^{n-1})du$.

Rule 3: If $y = u + v$, then $dy = du + dv$.

Rule 4: If $y = uv$, then $dy = vdu + udv$.

Rule 5: If $y = \frac{u}{v}$, then $dy = \frac{v\,du - u\,dv}{(dv)^2}$.

Total Derivatives

The total derivative for $y = f(v(x), x)$ is given by

(3A.69) $$\frac{dy}{dx} = \frac{\partial f}{\partial v}\frac{dv}{dx} + \frac{\partial f}{\partial x} = f_v\frac{dv}{dx} + f_x.$$

Similarly, for the function $y = f(x_1(t), x_2(t), t)$, the total derivative is given by

(3A.70) $$\frac{dy}{dt} = f_{x_1}\frac{dx_1}{dt} + f_{x_2}\frac{dx_2}{dt} + f_t.$$

Implicit Functions

Consider the function $g(x,y) = 0$. We wish to know whether there is a functional relationship between y and x (i.e., $y = y(x)$) over certain ranges and domains. The issue is addressed by the implicit function theorem (IFT).

Implicit Function Theorem for a Single Equation

Consider the function $F(y, x_1, x_2, \ldots, x_n) = 0$. If the partial derivatives $F_y, F_1, F_2, \ldots, F_n$ exist with $F_y \neq 0$ at the point being evaluated, then there is an implicit functional relationship $y = f(x_1, x_2, \ldots, x_n)$ with partial derivatives $\frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \ldots, \frac{\partial y}{\partial x_n}$.

For example, consider the function

(3A.71) $x^2 + y^2 = 9.$

The total differential of (3A.71) is given by

(3A.72) $2x\,dx + 2y\,dy = 0.$

Let $F_x = 2x$ and $F_y = 2y$. Note that $F_y \neq 0$ if $y \neq 0$. By the IFT,

(3A.73) $\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{2x}{2y} = -\frac{x}{y}$ if $y \neq 0$.

Consider a second example in which

(3A.74) $x^2 + y^2 + 6y = 9.$

Therefore, $F_x = 2x$ and $F_y = 2y + 6 \neq 0$ if $y \neq -3$. By the IFT,

(3A.75) $\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{2x}{(2y+6)} = -\frac{x}{(y+3)}$ if $y \neq -3$.

An Economic Application of the Implicit Function Theorem

Assume that a production function consists of two inputs (x_1 and x_2) and two outputs (y_1 and y_2) such that

(3A.76) $F(y_1, y_2, x_1, x_2) = 0.$

Taking the total differential of (3A.76) yields

(3A.77) $F_{y_1}dy_1 + F_{y_2}dy_2 + F_{x_1}dx_1 + F_{x_2}dx_2 = 0.$

By the IFT, we can hold any two variables constant (e.g., y_2 and x_1) and obtain the partial derivative of the remaining pair of variables as

(3A.78) $\frac{\partial y_1}{\partial x_2} = -\frac{F_{x_2}}{F_{y_1}}$ if $F_{y_1} \neq 0.$

Implicit Function Theorem for a System of Equations

Consider a system of equations:

$$(3A.79)\quad \begin{aligned} &F^1(y_1, y_2, \dots, y_m, x_1, x_2, \dots, x_n) = 0 \\ &F^2(y_1, y_2, \dots, y_m, x_1, x_2, \dots, x_n) = 0 \\ &\qquad\vdots \\ &F^m(y_1, y_2, \dots, y_m, x_1, x_2, \dots, x_n) = 0. \end{aligned}$$

It is useful to find the conditions under which, at least implicitly, the following functions exist:

$$(3A.80)\quad \begin{aligned} &y_1 = f^1(x_1, x_2, \dots, x_n) \\ &y_2 = f^2(x_1, x_2, \dots, x_n) \\ &\qquad\vdots \\ &y_m = f^m(x_1, x_2, \dots, x_n). \end{aligned}$$

We first take the total differential of (3A.79) and rearrange to obtain

$$(3A.81)\quad \begin{aligned} &F^1_{y_1} dy_1 + F^1_{y_2} dy_2 + \cdots + F^1_{y_m} dy_m \\ &\qquad = -(F^1_{x_1} dx_1 + F^1_{x_2} dx_2 + \cdots + F^1_{x_n} dx_n) \\ &F^2_{y_1} dy_1 + F^2_{y_2} dy_2 + \cdots + F^2_{y_m} dy_m \\ &\qquad = -(F^2_{x_1} dx_1 + F^2_{x_2} dx_2 + \cdots + F^2_{x_n} dx_n) \\ &\qquad\qquad\vdots \\ &F^m_{y_1} dy_1 + F^m_{y_2} dy_2 + \cdots + F^m_{y_m} dy_m \\ &\qquad = -(F^m_{x_1} dx_1 + F^m_{x_2} dx_2 + \cdots + F^m_{x_n} dx_n). \end{aligned}$$

We now define the matrices

$$\boldsymbol{F_y} = \left[\frac{\partial F^i}{\partial y_j}\right], i = 1, 2, \ldots, \text{m}; j = 1, 2, \ldots, n$$

$$\boldsymbol{dy} = [dy_1\ dy_2 \ldots dy_m]'$$

$$\boldsymbol{F_x} = \left[\frac{\partial F^i}{\partial x_j}\right], i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$$

and

$$\boldsymbol{dx} = [dx_1\ dx_2 \ldots dx_n]',$$

which allows (3A.81) to be compactly written as

(3A.82) $$\boldsymbol{F_y}\,\boldsymbol{dy} = -\boldsymbol{F_x}\,\boldsymbol{dx}.$$

If $|\boldsymbol{F_y}| \neq 0$, then $\boldsymbol{F_y^{-1}}$ exists and we can write

(3A.83) $$\boldsymbol{dy} = -\boldsymbol{F_y^{-1}}\boldsymbol{F_x}\,\boldsymbol{dx}.$$

Therefore, we have shown that the endogenous variables $\boldsymbol{y}$ can be expressed as implicit functions of the exogenous variables $\boldsymbol{x}$ such that

(3A.84)
$$y_1 = f^1(\boldsymbol{x})$$
$$y_2 = f^2(\boldsymbol{x})$$
$$\vdots$$
$$y_m = f^m(\boldsymbol{x}).$$

We can also recover the partial derivatives $\frac{\partial y_i}{\partial x_j}$ from (3A.83) by allowing $dx_p = 0$ for all $p \neq j$ such that

$$\text{(3A.85)}\quad \begin{bmatrix} dy_1 \\ . \\ . \\ dy_i \\ . \\ . \\ dy_m \end{bmatrix} = -\boldsymbol{F}_{\boldsymbol{y}}^{-1}\boldsymbol{F}_{\boldsymbol{x}} \begin{bmatrix} 0 \\ . \\ . \\ dx_j \\ . \\ . \\ 0 \end{bmatrix} = -\boldsymbol{F}_{\boldsymbol{y}}^{-1}\boldsymbol{F}_{\boldsymbol{x}} \begin{bmatrix} 0 \\ . \\ 0 \\ 1 \\ 0 \\ . \\ 0 \end{bmatrix} dx_j = -\boldsymbol{F}_{\boldsymbol{y}}^{-1}\boldsymbol{F}_{\boldsymbol{x}}\, \boldsymbol{e}_j\, dx_j,$$

where $\boldsymbol{e}_j$ is an elementary column vector consisting of a 1 in the j^{th} row and 0s elsewhere, and dx_j is the differential of x_j. Because dx_j is a non-zero scalar, we can divide both sides of (3A.85) by dx_j to obtain

$$\text{(3A.86)}\quad \left[\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}_j}\right] \equiv \begin{bmatrix} \frac{\partial y_1}{\partial x_j} \\ . \\ . \\ \frac{\partial y_i}{\partial x_j} \\ . \\ . \\ \frac{\partial y_m}{\partial x_j} \end{bmatrix} = -\boldsymbol{F}_{\boldsymbol{y}}^{-1}\boldsymbol{F}_{\boldsymbol{x}}\, \boldsymbol{e}_j.$$

If we wish to examine the ij^{th} partial derivative, we can premultiply (3A.86) by the elementary row vector $\boldsymbol{e}^i$, which selects the i^{th} row of the matrix

$$\text{(3A.87)}\quad \left[\frac{\partial y_i}{\partial x_j}\right] \equiv \boldsymbol{e}^i \begin{bmatrix} \frac{\partial y_1}{\partial x_j} \\ . \\ . \\ \frac{\partial y_i}{\partial x_j} \\ . \\ . \\ \frac{\partial y_m}{\partial x_j} \end{bmatrix} = -\boldsymbol{e}^i\boldsymbol{F}_{\boldsymbol{y}}^{-1}\boldsymbol{F}_{\boldsymbol{x}}\, \boldsymbol{e}_j\ .$$

»Chapter Four

ECONOMIC MODELING IN PRIMAL AND DUAL ENVIRONMENTS

Much regulatory policy that affects economic activity involves imposing constraints or taxes on consumers or production firms. In cases where regulatory constraints or taxes are imposed on consumers, the effects can often be modeled through impacts on producers. Supply functions are derived from firms' optimizing behavior subject to production functions and input prices. Demands for factors of production are derived from both consumers' desire for goods and services and producers' attempts to meet those desires through production activities. Researchers use equilibrium displacement models (EDMs) to connect economic activity among production and consumption sectors.

Economic activity can be modeled using a primal approach to firm-level profit maximization within a competitive environment in which producers are price takers. The assumption of a perfectly competitive market structure can be relaxed while constructing EDMs. This issue is addressed in Chapter 6. The current chapter illustrates the process of converting the basic primal problem into its dual structure. The conversion is necessary because of uncertainty regarding the functional form of production functions. In addition, it is usually the case that data required to estimate production functions are simply not available. Conversely, the dual modeling approach is much less data intensive as it only requires more readily available and commonly estimated parameters such as own-price and cross-price elasticities, factor shares, and (although less common) elasticities of input substitution.

A General Economic Model

Consider the profit-maximizing actions of a firm that operates in a perfectly competitive environment as an example of a primal approach. The firm's profit function is given by

$$\Pi = pf(\boldsymbol{x}) - \boldsymbol{w}'\boldsymbol{x}, \tag{4.1}$$

where Π is profit, p is the price of the firm's output, $f(\boldsymbol{x})$ is the firm's production function, $\boldsymbol{w}$ is a vector of input prices, and $\boldsymbol{x}$ is a vector of input quantities.

The profit-maximizing solution to a firm's decision process is found by obtaining first-order conditions (FOCs). FOCs are obtained by partially differentiating (4.1) with respect to each input and setting those equations equal to 0, such that

$$\boldsymbol{\Pi_x} = p\boldsymbol{f_x} - \boldsymbol{w} = \boldsymbol{0}. \tag{4.2}$$

The goal is to find the optimal level of each input ($\boldsymbol{x}^*$) that solves these equations. The Hessian matrix associated with the second-order conditions (SOCs) to the problem determines whether a unique solution exists in some neighborhood of input usage and, if it exists, whether the solution is a local minimum or a local maximum. If a solution exists, then SOCs must be checked to determine whether a profit maximizing or minimizing solution has been found.

The SOCs are represented by the following Hessian matrix:

$$\boldsymbol{\Pi_{xx}} = \begin{bmatrix} pf_{11} & pf_{12} & \cdot & \cdot & \cdot & pf_{1n} \\ pf_{21} & pf_{22} & \cdot & \cdot & \cdot & pf_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ pf_{n1} & pf_{n2} & \cdot & \cdot & \cdot & pf_{nn} \end{bmatrix} = p[f_{ij}] = p\boldsymbol{H}. \tag{4.3}$$

If $\boldsymbol{\Pi_{xx}}$ evaluated at point $\boldsymbol{x}^*$ is negative semi-definite, then the solution obtained from the FOCs is a local maximum. The Hessian in (4.3) is the Jacobian of the system presented in (4.2) and $\boldsymbol{H}$ is the Hessian of the production function $f(\boldsymbol{x})$. If the Jacobian matrix of (4.2) has a nonzero determinant, then the implicit function theorem (IFT) is used to determine whether implicit unique solutions exist. Consequently, if implicit input demand equations and output supply functions exist, we can write their general expressions as

$$\boldsymbol{x}^* = \boldsymbol{x}^*(p, \boldsymbol{w}) \tag{4.4}$$

$$q^* = q^*(p, \boldsymbol{w}). \tag{4.5}$$

Equations (4.4) and (4.5) have unique solutions only if the short run is being considered. That is, if at least one input is held constant, then unique solutions exist regarding the amounts of inputs used and output produced.

If $\boldsymbol{H}$ is negative definite, then $\boldsymbol{z'Hz} < 0$ for all nonzero vectors $\boldsymbol{z}$ and the comparative statics of the system can be shown to give

(4.6) $$\frac{\partial x_i^*}{\partial w_i} < 0$$

(4.7) $$\frac{\partial q^*}{\partial p} > 0.$$

These comparative static results are only relevant for the short run. They provide a constraint on the profit-maximizing problem such that it becomes a constrained, rather than an unconstrained, optimization problem. It also assumes that $\boldsymbol{H}$ is concave. However, in the long run, it has been well established that production functions are homogeneous of degree 1 (HD1) in inputs,[2] such that

(4.8) $$f(t\boldsymbol{x}) = t^1 f(\boldsymbol{x}).$$

The implication is that production functions can be scaled to any level of output when fixed inputs do not exist. Thus, if a production function is HD1, its gradient vector is homogeneous of degree 0 (HD0), which implies $\boldsymbol{H}(\boldsymbol{x}) \cdot \boldsymbol{x} = 0$ for all $\boldsymbol{x}$ and the matrix $\boldsymbol{H}^{-1}$ does not exist. Therefore, an infinite number of solutions exist for $\boldsymbol{x}^*$ and q^* because a unique firm size cannot be determined.

Consequently, if a production function is HD1 in inputs and/or we are interested in long-run solutions, then a constraint must be added to the system to find unique solutions. In some cases, the additional constraint may be the result of a factor of production whose supply is not infinitely elastic in the long run. For many other economic problems, the additional constraint is based on consumer demand for a firm's output. That is, consumer demand places a constraint on firms' output decisions because of cost-minimizing behavior.

Hence, (4.2) can be augmented such that

(4.9) $$q^D = q^D(p^D)$$

(4.10) $$q^S = f(\boldsymbol{x}^D)$$

2 Beattie, Taylor, and Watts (2009) show that any short-run production function in which one or more input factors are fixed can be converted to a long-run production function that is HD1 in inputs. In addition, production function homogeneity is completely consistent with constant, increasing, or decreasing cost industries. The implication of each is represented by the shape of the industry's long-run average total cost curve. The slope of this curve depends on the impacts of industry expansion on input costs and is not related to an underlying production function's technological homogeneity in terms of the use of those inputs.

(4.11) $p^S \boldsymbol{f_x} - \boldsymbol{w}^D = 0$

(4.12) $\boldsymbol{x}^S = \boldsymbol{x}^S(\boldsymbol{w}^S),$

where q^D is the quantity demanded for the good or service, p^D is the consumer demand price for a good or service, q^S is the quantity supplied of the good or service, $\boldsymbol{x}^D$ is the demand for inputs $\boldsymbol{x}$, p^S is the producer supply price for a good or service, $\boldsymbol{w}^D$ is the firm's demand prices for inputs $\boldsymbol{x}$, $\boldsymbol{x}^S$ is the supply of inputs $\boldsymbol{x}$, and $\boldsymbol{w}^S$ is the supply prices of inputs $\boldsymbol{x}$. In equilibrium and assuming competitive markets,

(4.13) $p^D = p^S = p$

(4.14) $q^D = q^S = q$

(4.15) $\boldsymbol{w}^D = \boldsymbol{w}^S = \boldsymbol{w}$

(4.16) $\boldsymbol{x}^D = \boldsymbol{x}^S = \boldsymbol{x}.$

Substituting (4.13)–(4.16) into (4.9)–(4.12), the system of equations becomes

(4.17) $q = q(p)$

(4.18) $q = f(\boldsymbol{x})$

(4.19) $p\boldsymbol{f_x}(\boldsymbol{x}) - \boldsymbol{w} = 0$

(4.20) $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{w}).$

System (4.17)–(4.20) consists of $2 + 2n$ equations with $2 + 2n$ endogenous variables p, q, $\boldsymbol{w}$, and $\boldsymbol{x}$ where n is the number of inputs. We can use various subsets of this system to obtain functional, and possibly only implicit, relationships between one or more of the variables and all other system variables if the conditions for the IFT hold. As noted previously, (4.19) cannot be uniquely solved for $\boldsymbol{x} = \boldsymbol{x}^*(p, \boldsymbol{w})$ when the production function is HD1 because the production function's Hessian is singular. However, when (4.18) and (4.19) are imposed simultaneously, the IFT demonstrates the existence of unique input demand functions given prices p and $\boldsymbol{w}$:

(4.21) $\boldsymbol{x}^* = \boldsymbol{x}^*(p, \boldsymbol{w})$

and an output supply equation

(4.22) $q^* = q^*(p, \boldsymbol{w})$.

Expressions (4.21) and (4.22) can be viewed as traditional input derived demand and output supply responses conditional upon a vector of output and input prices. These results are commonly used to conceptually develop traditional other-prices-held-constant (OPHC) quantity and own-price demand and supply schedules.

The derivation of explicit functional relationships in the above example requires the use of specific production functions. However, specifying and estimating production functions is both difficult and, often, intractable. Nonetheless, if we could explicitly derive mathematical expressions for (4.21) and (4.22), we could hold other prices constant and plot actual OPHC schedules $x_i^* = x_i^*(w_i|p, \boldsymbol{w}_j)$ and $y^* = y^*(p|\boldsymbol{w})$.

Although OPHC schedules are useful for developing economic intuition, they are generally avoided when developing EDMs. EDMs are often most useful for estimating endogenous price-quantity responses while allowing most or all prices included in the model to simultaneously adjust in response to policy or other external shocks. The figures presented in this chapter represent stylized total-response relationships as discussed by Buse (1958), Cochrane (1955), and Tomek and Robinson (1990). We use the expression "equilibrium trajectories" to denote total-response changes in prices and quantities induced by an exogenous shock. An equilibrium trajectory is a path that represents the movement from one price or quantity equilibrium to another. Total-response functions and equilibria trajectories are especially useful because estimated changes in consumer, producer, and input supply surplus metrics can be identified through integration. Silberberg (1990) noted, however, that such metrics cannot be obtained by integrating expressions (4.21) and (4.22).

The IFT allows us to implicitly identify total-response relationships by supplementing (4.17)–(4.20) with additional policy-induced equations and variables or by considering subsets of the system. For example, if we simultaneously impose (4.18), (4.19), and (4.20), the resulting system has $1 + 2n$ equations and $2 + 2n$ variables. If we then select one of the variables, say p, as exogenous to (4.18), (4.17), and (4.20), the IFT implies that we can implicitly identify the total-response functions:

(4.23) $q^* = q^*(p),\ \boldsymbol{x}^* = \boldsymbol{x}^*(p)$ and $\boldsymbol{w}^* = \boldsymbol{w}^*(p)$.

In this case, $q^*(p)$ can be viewed as the supply or total-response function of output with respect to changes in p while accounting for price and quantity feedback effects between producer first-order responses and input markets. Similarly, if we impose (4.17)–(4.20) while deleting one of the input supply schedules, say input x_j, from (4.20) and select input price w_j as exogenous, we can implicitly derive functions such as $x_j^* = x_j^*(w_j) = x_j^D(w_j)$, where $x_j^D(w_j)$ is the total-response derived demand for input j while accounting for endogenous interactions between consumers, producers, and other input markets. Consequently, the figures used in this and the following chapters present total-response relationships such as

(4.24) $q^* = q^*(p), p^* = p^*(q),$ and $w_i^D = w_i^D(x_i).$

Before proceeding, we note that explicitly recovering mathematical expressions for total-response functions and equilibria trajectories requires a knowledge of actual production functions and exogenously determined demand and input supply functions. We are seldom able to accurately identify these functions and estimate their associated parameters because such efforts are not only complicated, but also present other econometric, data, and practical problems. Many of these problems are also encountered when estimating production functions using data obtained from laboratory experiments. Even if such functions were known, inverting the components of system (4.17)–(4.20) and recovering the nonlinear explicit solutions for the primal total-response functions and equilibria trajectories would likely be mathematically intractable. The EDM approach is powerful because it allows for the relatively simple recovery of linear approximations to total-response functions and equilibria trajectories using more readily available elasticity estimates, factor share information, and linear algebra.

The EDM Dual-Based Approach

We obtain an EDM approximate solution to the above problem by mapping (4.10) and (4.11) into an equivalent dual system. The resulting system can be parameterized using estimates of commonly available economic variables, including factor shares, price elasticities of output demand, price elasticities of input supply, and elasticities of substitution between inputs within production technologies. Appendix 4A shows that the primal problem presented in (4.9)–(4.12) can be written as the following dual EDM:

(4.25) $E(q^D) = \eta^D E(p^D)$

(4.26) $E(p^S) = \sum_j K_j \, E(w_j^D)$

(4.27) $E(x_i^D) = E(q^S) + \sum_j K_j \, \sigma_{ij} E\big(w_j^D\big), \quad i = 1, 2, \dots, n$

(4.28) $E(x_i^s) = \sum_j \varepsilon_{ij} E\left(w_j^s\right),\;\; i = 1, 2, \dots, n$

where $E(\cdot) = \frac{d(\cdot)}{(\cdot)}$,

η^D is the own-price elasticity of demand, K_j are factor cost shares defined as

$K_j = \frac{w_j x_j^D}{\boldsymbol{w}\boldsymbol{x}^D}$,

σ_{ij} are Allen elasticities of substitution (AES), ε_{ij} are own- and cross-price elasticities of supply, and $\sum_j K_j = 1$ and $\sum_j K_j \sigma_{ij} = 0$ are conditions that must hold to maintain the system's theoretic consistency (Silberberg, 1990).[3]

An Application of the General Model Dual Approach

In the simplest cases, (4.25)–(4.28) can be used to model the effects of various exogenous shocks. For the moment, assume that output supply and demand prices and quantities are equal, as are input supply and demand prices and quantities. Consequently, exogenous shocks to this system result in new equilibrium prices and quantities without any policy-induced price or quantity wedges.[4] For ease of illustration, we assume that input supply quantities are a function of only own-input prices, such that $\varepsilon_{12} = \varepsilon_{21} = 0$.

Thus, a one-output, two-input EDM can be written as

(4.29) $E(q) = \eta^D E(p) + E(\theta_1)$

(4.30) $E(p) = K_1 E(w_1) + K_2 E(w_2) + E(\theta_2)$

(4.31) $E(x_1) = E(q) + K_1\sigma_{11} E(w_1) + K_2\sigma_{12} E(w_2) + E(\theta_3)$

(4.32) $E(x_2) = E(q) + K_1\sigma_{21} E(w_1) + K_2\sigma_{22} E(w_2) + E(\theta_4)$

(4.33) $E(x_1) = \varepsilon_1 E(w_1) + E(\theta_5)$

(4.34) $E(x_2) = \varepsilon_2 E(w_2) + E(\theta_6)$,

where θs represent exogenous shocks and $E(\cdot)$ are percentage changes. Equation (4.29) represents a consumer's total-response demand function and (4.30)–(4.32) represent a firm's production function and FOCs for the firm's profit-maximization

3 In the original primal form, equations (4.18) and (4.19) jointly represent the production technology and first-order optimizing behavioral conditions. We demonstrate in Appendix 4A that rearranging the joint total differential system of (4.18) and (4.19) results in the dual expressions (4.26) and (4.27). The latter jointly provide the output supply response and derived factor demands. The derived factor demands are decomposed into two components: (1) the output-held-constant substitution effects, and (2) the output effect. The expressions $K_i\sigma_{ij}$ in (4.27) are "output-held-constant" elasticities of derived demand for factor x_i with respect to a change in factor price w_j.

4 Such policies place wedges between specific variables of interest. This assumption is relaxed later in this chapter.

problem. Equations (4.33) and (4.34) are input supply functions. Equations (4.29)–(4.34) are considered behavioral equations because endogenous responses are assumed to result from economic agents' optimizing behavior. Exogenous shocks to any of these equations are modeled by percentage changes in θ_1 to θ_6.

Modeling Exogenous Shocks

Equations (4.29)–(4.34) represent an EDM that can be used to model several types of exogenous shocks. Specifically, the model is used to estimate changes in equilibria that result from positive or negative shocks to consumer demand, input supplies, and technological change. The EDM model presented in (4.29)–(4.34) is operationalized by moving the endogenous variables in those equations to the left-hand side to yield:

$$E(q) - \eta^D E(p) = E(\theta_1) \tag{4.35}$$

$$E(p) - K_1 E(w_1) - K_2 E(w_2) = E(\theta_2) \tag{4.36}$$

$$E(x_1) - E(q) - K_1\sigma_{11} E(w_1) - K_2\sigma_{12} E(w_2) = E(\theta_3) \tag{4.37}$$

$$E(x_2) - E(q) - K_1\sigma_{21} E(w_1) - K_2\sigma_{22} E(w_2) = E(\theta_4) \tag{4.38}$$

$$E(x_1) - \varepsilon_1 E(w_1) = E(\theta_5) \tag{4.39}$$

$$E(x_2) - \varepsilon_2 E(w_2) = E(\theta_6). \tag{4.40}$$

Using linear algebra, (4.35)–(4.40) can be written as

$$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -K_1 & -K_2 \\ -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} \\ -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} \\ 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \end{bmatrix}. \tag{4.41}$$

In a general form, (4.41) can be written as

(4.42) $\boldsymbol{Ay} = \boldsymbol{Bb},$

where $\boldsymbol{A}$ is a 6×6 matrix of parameters, $\boldsymbol{y}$ is a 6×1 vector of endogenous variables, $\boldsymbol{B}$ is a 6×6 diagonal matrix in which weights can be attached to any element of the $\boldsymbol{b}$ vector, and $\boldsymbol{b}$ is a 6×1 vector of exogenous shocks. If weights on the exogenous shocks are all equal to 1, then $\boldsymbol{B}$ is an identity matrix. For the following illustrations and throughout the remainder of this book, we assume that $\boldsymbol{B}$ is an identity matrix to simplify the exposition. Nonetheless, $\boldsymbol{B}$ can be used as a weighting mechanism for any of the elements in the $\boldsymbol{b}$ vector. After parameterizing the $\boldsymbol{A}$ matrix, the system's endogenous variables are solved for any exogenous shock $\boldsymbol{b}$ as:

(4.43) $\boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{b}.$

For the following examples, we use Gardner's (1988, p. 99) assumed parameterization for which the own-price elasticity of demand, η^D, is −0.60; the own-price elasticities of input supply (ε_1, ε_2) are 0.20 and 1.0, respectively; and factor shares K_1 and K_2 are equal to 0.30 and 0.70. We assume that the AES are $\sigma_{12} = \sigma_{21} = 1.0$. Although the terms σ_{11} and σ_{22} have no meaning as elasticities of substitution, they must be included in the model if the production technology is to be homogeneous of degree 0 (HD0) in all input and output prices and allow for the system to add up (Silberberg, 1990). These values are calculated as

$$\sigma_{11} = -\frac{K_2\sigma_{12}}{K_1} = -\frac{(0.70)(1.0)}{0.30} = -2.33 \text{ and}$$

$$\sigma_{22} = -\frac{K_1\sigma_{21}}{K_2} = -\frac{(0.30)(1.0)}{0.70} = -0.429.$$

Shocks to Consumer Demand and Equilibrium Trajectories

The effects of an exogenous shock to consumer demand can be modeled by allowing θ_1 to be nonzero. Figure 4.1 presents initial consumer demand D_0 and producer supply S_0 functions. A positive exogenous shock to consumer demand in the amount of θ_1 horizontally shifts the demand function from D_0 to D_1. The positive change in θ_1 causes a horizontal, rather than a vertical, shift of the demand function in Figure 4.1 because (4.29) is mathematically presented as an ordinary demand equation. In response, the equilibrium price and quantity would increase along the total-response supply curve, S_0, to p_1 and q_1. However, EDMs do not trace out this exact change in

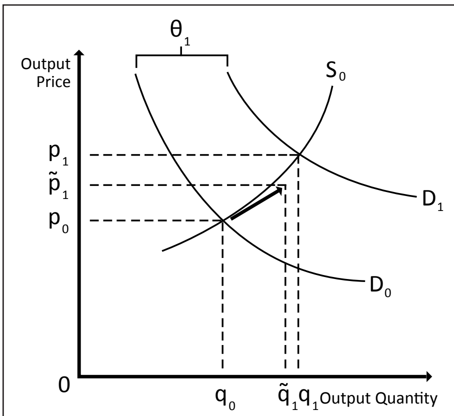

Figure 4.1. Exogenous Shock to Consumer Total-Response Demand

supply curve, S_0, to p_1 and q_1. However, EDMs do not trace out this exact change in equilibria. Further, unless the functional forms of the demand and supply functions depicted in Figure 4.1 are known with certainty (which is rare), other economic models are also unable to trace out this exact movement. An EDM estimates a linear equilibrium trajectory between the initial equilibrium and the new calculated equilibrium by moving along a line tangent to S_0 at the initial equilibria point (p_0, q_0) as noted by the arrow in Figure 4.1. Hence, the new equilibrium that results from an EDM is given by $\tilde{p}_1$ and $\tilde{q}_1$.

To model an exogenous shock to demand, we parameterize (4.41) using the previously noted values. For a 10% increase in consumer demand, $E(\theta_1) = 0.10$, the model is written as

$$\text{(4.44)} \quad \begin{bmatrix} 1 & 0.60 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.30 & -0.70 \\ -1 & 0 & 1 & 0 & 0.70 & -0.70 \\ -1 & 0 & 0 & 1 & -0.30 & 0.30 \\ 0 & 0 & 1 & 0 & -0.20 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1.0 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} 0.10 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}.$$

Equation (4.44) is solved using (4.43), which results in

$$\text{(4.45)} \quad \boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} 0.053 \\ 0.079 \\ 0.022 \\ 0.066 \\ 0.110 \\ 0.066 \end{bmatrix}.$$

The results show that the 10% increase in demand causes output, $E(q)$, to increase by 5.3% and output price, $E(p)$, to increase by 7.9%. The quantity of input 1, $E(x_1)$, increases by 2.2% while its price, $E(w_1)$, increases by 11.0%. The quantity of input 2, $E(x_2)$, increases by 6.6% while its price, $E(w_2)$, also increases by 6.6% because of the assumed unitary own-price elasticity of supply for input 2.

Shocks to Demand Assuming Fixed Input Proportions

The previous example explicitly considers variable input proportions between inputs 1 and 2 because the AES are assumed to be nonzero (Gardner, 1988). However, the results are altered if one imposes fixed input proportions such that $\sigma_{12} = \sigma_{21} = 0$ and, consequently, $\sigma_{11} = \sigma_{22} = 0$.

Consider an exogenous 10% increase in the demand for output such that demand increases from D_0 to D_1, as illustrated in Figure 4.1. To model an exogenous shock to demand assuming fixed input proportions, we parameterize (4.41) using the above estimates and with all AES equal to 0. Assuming a 10% increase in consumer demand, $E(\theta_1) = 0.10$, the EDM becomes

$$(4.46)\quad \begin{bmatrix} 1 & 0.60 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.30 & -0.70 \\ -1 & 0 & 1 & 0 & 0.00 & 0.00 \\ -1 & 0 & 0 & 1 & 0.00 & 0.00 \\ 0 & 0 & 1 & 0 & -0.20 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1.0 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} 0.10 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}.$$

Equation (4.46) is solved as in (4.43), yielding

$$(4.47)\quad \boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} 0.043 \\ 0.095 \\ 0.043 \\ 0.043 \\ 0.216 \\ 0.043 \end{bmatrix}.$$

The results indicate that output, $E(q)$, increases by 4.3%, which is smaller than the variable input proportions case, while the equilibrium price, $E(p)$, increases by 9.5%, which is larger than that obtained for the variable input proportions case. The quantity of input 1, $E(x_1)$, increases by 4.3% and its price, $E(w_1)$, increases by 21.6%. Both percentage changes are almost twice as large as in the variable input proportions case in (4.45). Because of the assumed fixed input proportion technology, the use of input 2, $E(x_2)$, increases by 4.3%, which is less than the variable input proportions case in (4.45), while its price, $E(w_2)$, increases by the same 4.3% because of the assumed unitary own-price elasticity of supply for input 2. These results occur because fixed input proportions restrict production flexibility and result in a more muted output response to the demand increase.

An Exogenous Shock to Input Supplies

The EDM presented in (4.35)–(4.40) can be used to estimate the effects of an exogenous shock to the supply of either or both inputs by allowing θ_5 and/or θ_6 to be nonzero. For example, consider a case in which a 10% exogenous decrease in the supply of input 1 occurs. Figure 4.2 presents total-response derived demand, D_0, and supply, S_0, curves for input 1. A decrease in the supply of input 1 is modeled as a horizontal shift to the left from S_0 to S_1 in Figure 4.2. Again, the change is illustrated by a horizontal shift in the supply function because (4.33) is presented as an ordinary supply function. The input price increases to w_1 while the use of input 1 declines to x_1. Note that the EDM does not trace out the actual equilibrium trajectory along the nonlinear total-response derived demand curve D_0 but rather provides a linear approximation (the arrow in Figure 4.2) to the actual equilibrium by moving along the line tangent to D_0 at the initial equilibrium (x_0,w_0). We note that the functional form of D_0 is always unknown, but the EDM provides a linear approximation to the true equilibrium trajectory (as indicated by the arrow in Figure 4.2).

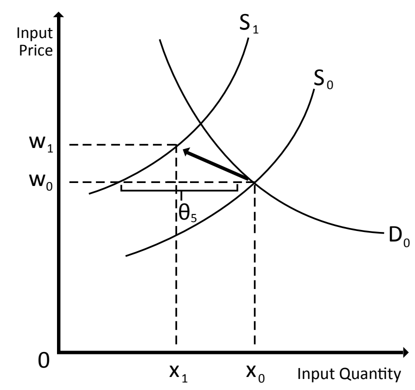

Figure 4.2. Exogenous Shock to Input 1

To find this new equilibrium point, the exogenous shock is entered in vector ***b*** of (4.41) as $E(\theta_5) = -0.10$ while setting all other values in the vector equal to 0. This entry indicates that the ordinary supply function for input 1 shifts to the left and the supply schedule decreases by 10% for any given input price, w, from the initial equilibrium level. Returning to the variable input proportions scenario, (4.44) now becomes

$$(4.48) \quad \begin{bmatrix} 1 & 0.60 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.30 & -0.70 \\ -1 & 0 & 1 & 0 & 0.70 & -0.70 \\ -1 & 0 & 0 & 1 & -0.30 & 0.30 \\ 0 & 0 & 1 & 0 & -0.20 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1.0 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ -0.10 \\ 0.0 \end{bmatrix}.$$

The solution for the endogenous variables in (4.48) is indicated by the values in vector ***y*** as

$$(4.49) \quad \boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} -0.020 \\ 0.033 \\ -0.081 \\ 0.007 \\ 0.094 \\ 0.007 \end{bmatrix}.$$

The exogenous shock causes the use of input 1, $E(x_1)$, to decline by 8.1% and its price, $E(w_1)$, to increase by 9.4%. Because input 2 is a substitute for input 1, the quantity of input 2 used, $E(x_2)$, increases by 0.7%, and its price, $E(w_2)$, increases by 0.7%. The magnitude of the quantity and price impacts on input 2 are identical because of the assumed unitary elasticity of supply for input 2. The exogenous shock causes the output supply function to shift to the left. Hence, the equilibrium output price, $E(p)$, increases by 3.3% while output quantity, $E(q)$, declines by 2%.

More Than One Exogenous Shock

An EDM can be used to evaluate changes in equilibria for simultaneous exogenous shocks to a system. For example, the COVID-19 pandemic caused multiple disruptions to the food distribution system. Almost one-half of U.S food consumption consists of food consumed away-from-home, but the pandemic caused many restaurants to close or curtail operations. Some of these changes were the result of government mandates, while others resulted from reductions in travel and entertainment activities caused by health concerns. Concurrently, the supply of labor decreased as the labor market contracted and allowances for slower production lines and/or worker safety measures occurred.

The EDM presented in (4.44) can be used to evaluate multiple exogenous shocks to an economic system. Hence, EDMs are more flexible than traditional comparative static approaches. For example, assume that the COVID-19 pandemic caused a 10% reduction in the demand for restaurant meals because of reduced travel and simultaneously decreased the supply of labor, x_1, by 10%.

The EDM is operationalized for this problem by setting $E(\theta_1)$ equal to −0.10 and $E(\theta_5)$ equal to −0.10 in vector $\boldsymbol{b}$, with all other values set equal to 0. These values reflect a 10% decrease in the demand for restaurant meals and a 10% decrease in the quantity of labor supplied at a given wage or input price. Solving for the endogenous variables as noted in (4.43) yields

$$(4.50)\qquad \boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} -0.072 \\ -0.046 \\ -0.103 \\ -0.059 \\ -0.015 \\ -0.059 \end{bmatrix}.$$

The results indicate that the provision of restaurant meals by the service sector, $E(q)$, declines by 7.2% while the price of meals, $E(p)$, declines by 4.6%. Note that the reduction in meals provided by the restaurant sector reduces the demand for labor, causing the quantity of labor used, $E(x_1)$, to decline by 10.3%. Although the supply of labor has decreased, the reduced demand for meals causes the equilibrium price of labor, $E(w_1)$, to decline by 1.5%. In addition, the use of the second input, $E(x_2)$, say food inputs, declines by 5.9% because of the reduced demand for restaurant meals. The price of food inputs, $E(w_2)$, also declines by 5.9%.

It is possible that the price of labor could increase if the demand for restaurant meals declined by a smaller amount, even though the demand for labor declines. This could happen if the decrease in the supply of labor more than offsets the decrease in demand for labor. For example, suppose that the demand for restaurant meals declined by only 5% due to the COVID-19 pandemic while the supply of labor decreased by 10%. Equation (4.44) is operationalized by setting $E(\theta_1)$ equal to −0.05 and $E(\theta_5)$ equal to −0.10 in vector $\boldsymbol{b}$, with all other values set equal to 0. Solving for the endogenous variables in (4.43) yields

$$\boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} -0.046 \\ -0.007 \\ -0.092 \\ -0.026 \\ 0.039 \\ -0.026 \end{bmatrix}. \tag{4.51}$$

In this case, the quantity of meals, $E(q)$, supplied by the service sector declines by 4.6% while the price of meals, $E(p)$, declines by 0.7%. Although the quantity reduction is approximately one-third smaller than the example in (4.50), the reduction in price is much smaller than the previous estimate. The price of labor, $E(w_1)$, increases by 3.9% even as the use of labor, $E(x_1)$, declines by 9.2%. That is, the decrease in labor supply more than offsets the reduction in the demand for labor caused by the 5% reduction in demand for meals so that the equilibrium price of labor increases. In (4.50), the opposite was true, as the reduction in the demand for labor was greater than the decrease in labor supply, such that equilibrium labor prices declined. In addition, the decline in labor usage is similar to that in (4.50). Finally, the use of input 2, $E(x_2)$, declines by 2.6%, as does the price of input 2, $E(w_2)$. The decline in both values is much smaller than that in (4.50).

Comparing Models with Respect to Homogeneity of Degree 0 in All Prices

Many researchers have developed EDMs by directly considering the demand and supply for a good or service at one market level and the demand and supply of a major input that is produced at another market level (Brester, Marsh, and Atwood, 2009; Pendell et al., 2010, 2013). However, some models do not meet the theoretically consistent requirement that the EDM system of equations should be HD0 in all input and output prices.

To illustrate the bias caused by ignoring this condition, we construct an EDM model that does not meet this requirement and estimate changes in equilibria caused by an exogenous shock. Then, we develop an EDM based on the above dual derivation that is HD0 in all prices and use it to evaluate changes in equilibria caused by an identical exogenous shock. The exercise is used to illustrate the importance of constructing EDMs that are HD0 in all input and output prices.

An EDM That Is Not Homogeneous of Degree 0 in All Prices

Although EDMs have been developed using various approaches, a common method is to specify general demand and supply functions, which are then totally differentiated and converted to proportional elasticity forms. For example, consider the demand and supply of consumer retail beef products. The consumer demand for beef could be represented by

(4.52) $$q^D = f_1(p^D) + \varphi_1,$$

where q^D is the quantity of retail beef consumed and p^D is the consumer price of retail beef. Equation (4.52) represents the primary demand level (Tomek and Robinson, 1990).

Food processors slaughter and convert live cattle into finished beef products. The supply of finished beef products is given by

(4.53) $$q^S = f_2(p^S, x^D) + \varphi_2,$$

where q^S is the quantity of retail beef produced, p^S is the producer price of retail beef, and x^D is the quantity of live cattle demanded by processors. Thus, (4.53) represents a derived supply function (Tomek and Robinson, 1990).

Processors who manufacture beef products have a derived demand for live cattle produced by feedlot enterprises, which is the major input into producing retail beef. The derived demand is illustrated by

(4.54) $$x^D = f_3(w^D, q^D) + \varphi_3,$$

where w^D is the input demand price of live cattle. Finally, feedlot enterprises supply live cattle to the processing sector as indicated by

(4.55) $$x^S = f_4(w^S) + \varphi_4,$$

where x^S is the quantity of live cattle supplied by feedlots and w^S is the input supply price of live cattle. This supply function is often termed the primary supply function (Tomek and Robinson, 1990). In addition, the following equilibrium conditions are assumed:

(4.56) $$q^D = q^S = q$$

(4.57) $$p^D = p^S = p$$

(4.58) $x^D = x^S = x$

(4.59) $w^D = w^S = w,$

so that (4.52)–(4.55) is written as

(4.60) $q = f_1(p) + \varphi_1$

(4.61) $q = f_2(p, x) + \varphi_2$

(4.62) $x = f_3(w, q) + \varphi_3$

(4.63) $x = f_4(w) + \varphi_4.$

Totally differentiating (4.60)–(4.63) yields

(4.64) $dq = \frac{dq}{dp}dp + d\varphi_1$

(4.65) $dq = \frac{dq}{dp}dp + \frac{dq}{dx}dx + d\varphi_2$

(4.66) $dx = \frac{dx}{dw}dw + \frac{dx}{dq}dq + d\varphi_3$

(4.67) $dx = \frac{dx}{dw}dw + d\varphi_4.$

Dividing (4.64) and (4.65) by q and (4.66) and (4.67) by x yields

(4.68) $\frac{dq}{q} = \left(\frac{1}{q}\right)\frac{dq}{dp}dp + \left(\frac{1}{q}\right)d\varphi_1$

(4.69) $\frac{dq}{q} = \left(\frac{1}{q}\right)\frac{dq}{dp}dp + \left(\frac{1}{q}\right)\frac{dq}{dx}dx + \left(\frac{1}{q}\right)d\varphi_2$

(4.70) $\frac{dx}{x} = \left(\frac{1}{x}\right)\frac{dx}{dw}dw + \left(\frac{1}{x}\right)\frac{dx}{dq}dq + \left(\frac{1}{x}\right)d\varphi_3$

(4.71) $\frac{dx}{x} = \left(\frac{1}{x}\right)\frac{dx}{dw}dw + \left(\frac{1}{x}\right)d\varphi_4.$

Multiplying the first term on the right-hand side of (4.68) by $\frac{p}{p}$ results in

(4.72) $\frac{dq}{q} = \left(\frac{1}{q}\right)\left(\frac{p}{p}\right)\frac{dq}{dp}dp + \left(\frac{1}{q}\right)d\varphi_1.$

Rearranging yields

(4.73) $\frac{dq}{q} = \left(\frac{dq}{dp}\frac{p}{q}\right)\left(\frac{dp}{p}\right) + \frac{d\varphi_1}{q}$

or

(4.74) $E(q) = \eta^q E(p) + E(\varphi_1),$

where η^q is the own-price elasticity of demand for the retail product.

Next, multiply the first term on the right-hand side of (4.69) by $\frac{p}{p}$ and the second term by $\frac{x}{x}$ to yield

(4.75) $\frac{dq}{q} = \left(\frac{1}{q}\right)\left(\frac{p}{p}\right)\frac{dq}{dp}dp + \left(\frac{1}{q}\right)\left(\frac{x}{x}\right)\frac{dq}{dx}dx + \left(\frac{1}{q}\right)d\varphi_2.$

Rearranging yields

(4.76) $\frac{dq}{q} = \left(\frac{dq}{dp}\frac{p}{q}\right)\frac{dp}{p} + \left(\frac{dq}{dx}\frac{x}{q}\right)\frac{dx}{x} + \frac{d\varphi_2}{q}$

or

(4.77) $E(q) = \varepsilon^q E(p) + \tau E(x) + E(\varphi_2),$

where ε^q is the own-price derived supply elasticity of retail beef products and τ represents an elasticity of quantity transmission between the live cattle sector and the retail beef sector.

Next, multiply the first term on the right-hand side of (4.70) by $\frac{w}{w}$ and the second term by $\frac{q}{q}$, which yields

(4.78) $\frac{dx}{x} = \left(\frac{1}{x}\right)\left(\frac{w}{w}\right)\frac{dx}{dw}dw + \left(\frac{1}{x}\right)\left(\frac{q}{q}\right)\frac{dx}{dq}dq + \left(\frac{1}{x}\right)d\varphi_3.$

Rearranging (4.78) yields

(4.79) $\frac{dx}{x} = \left(\frac{dx}{dw}\frac{w}{x}\right)\frac{dw}{w} + \left(\frac{dx}{dq}\frac{q}{x}\right)\frac{dq}{q} + \frac{d\varphi_3}{x}$

or

(4.80) $E(x) = \eta^{x_1} E(w) + \gamma E(q) + E(\varphi_3),$

where η^{x_1} is the own-price derived demand elasticity for live cattle and γ represents an elasticity of quantity transmission between the retail beef sector and the live cattle sector.

Finally, multiply the first term on the right-hand side of (4.71) by $\frac{w}{w}$, which yields

(4.81) $$\frac{dx}{x} = \left(\frac{1}{x}\right)\left(\frac{w}{w}\right)\left(\frac{dx}{dw}\right)dw + \frac{d\varphi_4}{x}.$$

Rearranging yields

(4.82) $$\frac{dx}{x} = \left(\frac{dx}{dw}\frac{w}{x}\right)\frac{dw}{w} + \frac{d\varphi_4}{x}$$

or

(4.83) $$E(x) = \varepsilon^{x_1}E(w) + E(\varphi_4),$$

where ε^{x_1} is the own-price primary supply elasticity of live cattle.

Collecting (4.74), (4.77), (4.80), and (4.83) results in an EDM model of the form

(4.84) $$E(q) = \eta^q E(p) + E(\varphi_1)$$ primary retail demand

(4.85) $$E(q) = \varepsilon^q E(p) + \tau E(x) + E(\varphi_2)$$ derived retail supply

(4.86) $$E(x) = \eta^{x_1}E(w) + \gamma E(q) + E(\varphi_3)$$ derived feedlot demand

(4.87) $$E(x) = \varepsilon^{x_1}E(w) + E(\varphi_4).$$ primary feedlot supply

Moving the endogenous variables to the left-hand side yields

(4.88) $$E(q) - \eta^q E(p) = E(\varphi_1)$$

(4.89) $$E(q) - \varepsilon^q E(p) - \tau E(x) = E(\varphi_2)$$

(4.90) $$E(x) - \eta^{x_1}E(w) - \gamma E(q) = E(\varphi_3)$$

(4.91) $$E(x) - \varepsilon^{x_1}E(w) = E(\varphi_4).$$

In matrix notation, (4.88)–(4.91) are written as

$$(4.92)\quad \begin{bmatrix} 1 & -\eta^q & 0 & 0 \\ 1 & -\varepsilon^q & -\tau & 0 \\ -\gamma & 0 & 1 & -\eta^{x_1} \\ 0 & 0 & 1 & -\varepsilon^{x_1} \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x) \\ E(w) \end{bmatrix} = \begin{bmatrix} E(\varphi_1) \\ E(\varphi_2) \\ E(\varphi_3) \\ E(\varphi_4) \end{bmatrix}.$$

The parametrization of (4.92) requires estimates of η^q, ε^q, η^{x_1}, ε^{x_1}, τ, and γ. However, to illustrate the shortcomings of the non-HD0 model, assume that the values are $\eta^q = \eta^{x_1} = -1.0$ and $\varepsilon^q = \varepsilon^{x_1} = \tau = \gamma = 1.0$, such that

$$(4.93)\quad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x) \\ E(w) \end{bmatrix} = \begin{bmatrix} E(\varphi_1) \\ E(\varphi_2) \\ E(\varphi_3) \\ E(\varphi_4) \end{bmatrix}.$$

Note that there are no behavioral equations in the EDM that specify input price, w, or output price, p. Hence, it is not possible to conduct a direct test of the HD0 in all prices using (4.93). Although it is possible to include additional equations to perform such a test, it is sufficient to simply illustrate the shortcomings of the model caused by a lack of HD0 in all prices.

Assume that an exogenous shock decreases the supply of feeder cattle by 10%. This would be represented by setting $E(\varphi_4) = -0.10$. Solving (4.93) for the endogenous variables results in

$$(4.94)\quad \begin{bmatrix} E(q) \\ E(p) \\ E(x) \\ E(w) \end{bmatrix} = \begin{bmatrix} 0.667 & 0.667 & 0.333 & 0.333 \\ 0.333 & -0.667 & -0.333 & -0.333 \\ 0.333 & 0.333 & 0.667 & 0.667 \\ 0.333 & 0.333 & 0.667 & -0.333 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.10 \end{bmatrix}$$

or

$$(4.95)\quad \begin{bmatrix} E(q) \\ E(p) \\ E(x) \\ E(w) \end{bmatrix} = \begin{bmatrix} -0.033 \\ 0.033 \\ -0.067 \\ 0.033 \end{bmatrix}.$$

The results indicate that retail quantity, $E(q)$, declines by 3.3%, while retail prices, $E(p)$, increase by 3.3%. This is a consistent result given the assumed demand and supply elasticities of –1.0 and 1.0, respectively. The model indicates that the price of live cattle inputs, $E(w)$, increases by 3.3% but that the quantity of live cattle being produced, $E(x)$, declines by 6.7%. However, the assumed price, τ, and quantity, γ, transmission elasticities of 1.0 should cause the system to add up. That is, quantity changes at the feedlot level should be equivalent to quantity changes occurring at the consumer level if all values are represented by retail-weight equivalent metrics given that the model does not allow for variable input proportions between live cattle and other inputs. Clearly, the results in (4.95) cannot be correct given that the amount of live cattle being produced declines by 6.7% while the quantity of retail beef declines by only 3.3%.[5]

An EDM That Is Homogeneous of Degree 0 in All Prices

The EDM developed in (4.25)–(4.28) for a single output (retail beef) and two inputs (live cattle, x_1, and all other processing inputs, x_2) is written as

(4.96) $$E(q) = \eta^q E(p) + E(\theta_1)$$

(4.97) $$E(p) = K_1 E(w_1) + K_2 E(w_2) + E(\theta_2)$$

(4.98) $$E(x_1) = E(q) + K_1 \sigma_{11} E(w_1) + K_2 \sigma_{12} E(w_2) + E(\theta_3)$$

(4.99) $$E(x_2) = E(q) + K_1 \sigma_{21} E(w_1) + K_2 \sigma_{22} E(w_2) + E(\theta_4)$$

(4.100) $$E(x_1) = \varepsilon^{x_1} E(w_1) + E(\theta_5)$$

(4.101) $$E(x_2) = \varepsilon^{x_2} E(w_2) + E(\theta_6),$$

where q is the quantity demanded of retail beef; p is the retail price of beef; w_1 is the input price of live cattle; w_2 is the input price of all other processing inputs; η^q is the own-price elasticity of demand for retail beef; ε^{x_1} is the own-price elasticity of supply of cattle; ε^{x_2} is the own-price elasticity of supply of all other processing inputs; K_j represent factor shares, $K_j = \left(\frac{w_j x_j}{wx}\right)$, such that $\sum_j K_j = 1$; and σ_{ij} is the AES between inputs i and j. Silberberg (1990) notes that $\sum_j K_j \sigma_{ij} = 0$ is necessary to ensure that the system of equations is HD0 in input and output prices.

For the following example, we use the simplifying assumption that input supply quantities are functions of only their own-input prices rather than being influenced by the price of the other input in the system. This assumption is probably reasonable given that the impact of the price of all processing inputs, w_2, likely has an inconsequential influence on the supply of live cattle, x_1, and vice versa.

Equation (4.96) represents retail demand, (4.97)–(4.99) represent the production technologies derived from the first-order conditions for profit maximization, and (4.100) and (4.101) represent input supply functions. To test whether the

5 This is the example referenced in the preface. The next section presents the results using an EDM that is HD0 in all prices.

model presented in (4.96)–(4.101) is HD0 in input and output prices, we must alter the model by adding three equations that allow for price wedges to exist between demand and supply output and input prices:

(4.102) $E(q) = \eta^q E(p^D) + E(\theta_1)$

(4.103) $E(p^S) = K_1 E(w_1^D) + K_2 E(w_2^D) + E(\theta_2)$

(4.104) $E(x_1) = E(q) + K_1 \sigma_{11} E(w_1^D) + K_2 \sigma_{12} E(w_2^D) + E(\theta_3)$

(4.105) $E(x_2) = E(q) + K_1 \sigma_{21} E(w_1^D) + K_2 \sigma_{22} E(w_2^D) + E(\theta_4)$

(4.106) $E(x_1) = \varepsilon^{x_1} E(w_1^S) + E(\theta_5)$

(4.107) $E(x_2) = \varepsilon^{x_2} E(w_2^S) + E(\theta_6)$

(4.108) $E(p^D) = E(p^S) + E(\theta_7)$

(4.109) $E(w_1^D) = E(w_1^S) + E(\theta_8)$

(4.110) $E(w_2^D) = E(w_2^S) + E(\theta_9)$,

where the superscript *D* represents the demand price for retail beef or inputs and the superscript *S* represents the supply price for retail beef or inputs.

Equations (4.102)–(4.107) are behavioral equations, while (4.108)–(4.110) are equilibrium equations. Note that (4.108) allows for a price wedge to exist between the demand and supply price of retail beef, while (4.109) and (4.110) allow for price wedges between the demand and supply prices of the two inputs. In equilibrium and assuming the absence of taxes or subsidies, there would be no difference between demand and supply prices, which would make these equations superfluous. However, (4.108)–(4.110) allow us to test for HD0 across input and output prices.

The model is operationalized by moving the endogenous variables of (4.102)–(4.110) to the left-hand side:

(4.111) $E(q) - \eta^q E(p^D) = E(\theta_1)$

(4.112) $E(p^S) - K_1 E(w_1^D) - K_2 E(w_2^D) = E(\theta_2)$

(4.113) $E(x_1) - E(q) - K_1\sigma_{11}E(w_1^D) - K_2\sigma_{12}E(w_2^D) = E(\theta_3)$

(4.114) $E(x_2) - E(q) - K_1\sigma_{21}E(w_1^D) - K_2\sigma_{22}E(w_2^D) = E(\theta_4)$

(4.115) $E(x_1) - \varepsilon^{x_1}E(w_1^S) = E(\theta_5)$

(4.116) $E(x_2) - \varepsilon^{x_2}E(w_2^S) = E(\theta_6)$

(4.117) $E(p^D) - E(p^S) = E(\theta_7)$

(4.118) $E(w_1^D) - E(w_1^S) = E(\theta_8)$

(4.119) $E(w_2^D) - E(w_2^S) = E(\theta_9)$.

Using linear algebra, (4.111)–(4.119) are written as

(4.120) $\boldsymbol{Ay} = \boldsymbol{b},$

where $\boldsymbol{A}$ is a 9 × 9 matrix of parameters, $\boldsymbol{y}$ is a 9 × 1 vector of endogenous variables, and $\boldsymbol{b}$ is a 9 × 1 vector of exogenous shocks such that

(4.121)
$$\begin{bmatrix} 1 & -\eta^q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon^{x_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon^{x_2} \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p^D) \\ E(p^S) \\ E(x_1) \\ E(x_2) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \end{bmatrix}.$$

After parameterizing the $\boldsymbol{A}$ matrix, the system's endogenous variables are solved for any exogenous shock $\boldsymbol{b}$ as

(4.122) $\boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{b}.$

To compare the HD0 and non-HD0 examples, we parameterize the model by setting the retail demand elasticity equal to –1.0 and the input supply elasticities equal to 1.0. In addition, we set the elasticity of substitution equal to 0 ($\sigma_{12} = \sigma_{21} = 0.0$) and assume that the factor share of live cattle, K_1, equals 0.90 and the factor share of all other processing inputs, K_2, equals 0.10. As noted earlier, the terms

σ_{11} and σ_{22} have no meaning as elasticities of substitution but must be included in the model if the system of equations is to be HD0 in all prices. Hence, these values are calculated as

$$\sigma_{11} = -\frac{K_2\sigma_{12}}{K_1} = -\frac{(0.30)(0.0)}{0.90} = 0.0$$

$$\sigma_{22} = -\frac{K_1\sigma_{21}}{K_2} = -\frac{(0.90)(0.0)}{0.10} = 0.0.$$

Thus, the $\boldsymbol{A}$ matrix in (4.121) is parameterized as

$$(4.123)\quad \begin{bmatrix} 1 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -0.90 & -0.10 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0.0 & 0.0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0.0 & 0.0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1.0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1.0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}.$$

Within any production system, a scalar multiplication of all input and output prices should have no effect upon the quantities of inputs used or output produced, such that

$$(4.124)\quad p\boldsymbol{f_x} - \boldsymbol{w} = 0 \Rightarrow \boldsymbol{f_x} = \left(\frac{1}{p}\right)\boldsymbol{w} = \left(\frac{1}{tp}\right)t\boldsymbol{w} = t^o\left(\frac{1}{p}\right)\boldsymbol{w}$$

for any scalar t. This implies that equal percentage increases in input prices, coupled with an identical percentage increase in the producer output price, should not influence equilibrium output or input quantities. The issue is best illustrated by considering the profit function for a two-input production process:

$$(4.125)\quad \Pi = py(x_1, x_2) - w_1x_1 - w_2x_2.$$

The FOCs for profit maximization of (4.125) are given by

$$(4.126)\quad p\boldsymbol{f_x} - \boldsymbol{w} = 0.$$

If all input and output prices are multiplied by a scalar t, then the profit function becomes

$$(4.127)\quad (t)\Pi = (t)py(x_1, x_2) - (t)w_1x_1 - (t)w_2x_2$$

or

$$(t)\Pi = (t)(py(x_1, x_2) - w_1x_1 - w_2x_2). \tag{4.128}$$

The FOCs for (4.126) are given by

$$(t)(p\boldsymbol{f_x} - \boldsymbol{w}) = 0. \tag{4.129}$$

Consequently, because the profit function is HD1 with respect to all prices, the FOCs for profit maximization are HD0 in all prices. If a tax on inputs caused a doubling of input prices paid by a producer and a subsidy was placed on the output produced so that a producer received twice the price for their production, there would be no change in either input usage or production.

To test this for a doubling, or a 100% increase of input prices, allow $E(\theta_8)$ = 1.00 and $E(\theta_9)$ = 1.00 in vector $\boldsymbol{b}$ of (4.121). This represents a 100% tax on each input so that the demand price of each input is greater than its supply price. Therefore, the input price wedges are entered as positive numbers. Further, consider an output price subsidy of 100% that is simultaneously modeled by allowing $E(\theta_7)$ = –1.00. That is, the output supply price that producers receive is 100% larger than the output demand price that consumers pay. Essentially, this is a simple rescaling of the model's price variables. Thus, vector $\boldsymbol{b}$ in system (4.121) becomes

$$\boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1.00 \\ 1.00 \\ 1.00 \end{bmatrix}. \tag{4.130}$$

Using the matrices indicated in (4.123) and (4.130), the solution to (4.122) is

$$\boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p^D) \\ E(p^S) \\ E(x_1) \\ E(x_2) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.00 \\ 1.00 \\ 0.00 \\ 0.00 \\ 1.00 \\ 1.00 \\ 0.00 \\ 0.00 \end{bmatrix}. \tag{4.131}$$

The EDM results show that a doubling of input prices and the producer output price causes a 100% increase in input demand prices, $E(w_1^D)$ and $E(w_2^D)$, and a 100% increase in the price received by producers for output, $E(p^S)$. However, no changes in input use or production output occur. This is the expected and necessary result from economic theory for an EDM to be HD0 in all prices.

A 10% Shock to Input 1

Given that the EDM in (4.96)–(4.101) is HD0 in all prices, we now compare the results for a 10% decrease in the supply of live cattle to the results presented in (4.95). Recall that equilibrium conditions in (4.108)–(4.110) were added to the original EDM for the purpose of testing for HD0 in all prices. In addition to testing for homogeneity, the equations could be used to estimate the effects of various legislative policies (e.g., imposing taxes on inputs or an intervention in the output market). However, the equations are not needed to estimate the effects of a 10% decrease in the supply of input 1. For ease of illustration, consider the effects of the input supply decrease using only (4.111)–(4.116). After removing the superscripts and using (4.123), the matrix form of the EDM is given by

$$(4.132)\quad \begin{bmatrix} 1 & 1.00 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.90 & -0.10 \\ -1 & 0 & 1 & 0 & 0.00 & 0.00 \\ -1 & 0 & 0 & 1 & 0.00 & 0.00 \\ 0 & 0 & 1 & 0 & -1.00 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1.00 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \end{bmatrix}.$$

A 10% decrease in the supply of input 1, live cattle, is modeled as $E(\theta_5) = -0.10$ in (4.132). Thus, vector $\boldsymbol{b}$ becomes

$$(4.133)\quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.10 \\ 0 \end{bmatrix}.$$

Using the parameterization in (4.132) and the shock presented in (4.133), the solution for the endogenous variables in (4.132) results in

$$(4.134) \quad \boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} -0.045 \\ 0.045 \\ -0.045 \\ -0.045 \\ 0.055 \\ -0.045 \end{bmatrix}.$$

The equilibrium quantity of output at the retail level, $E(q)$, decreases by 4.5% and retail price $E(p)$ increases by 4.5%. The two values are identical in absolute value because of the assumed unitary own-price elasticity of demand. The model in (4.95), which was not HD0 in all prices, estimated a retail level quantity decline of 3.3% and a price increase of 3.3%.

Note that the 4.5% decline in the use of input 1, cattle, is identical to the decline in beef production. This is the expected result given the assumption of fixed input proportions. The previous non-HD0 EDM indicated that the reduction in cattle usage was twice as large as the reduction in beef output. The theoretically consistent EDM indicates that these reductions are equal given the assumed unitary supply and demand elasticities and fixed input proportions.

Note that the reduction in output, $E(q)$, and use of input 1, $E(x_1)$, are both equal to −4.5%. This result is generated by a 10% reduction in the supply of input 1. This shock was selected so that the results of the theoretically consistent EDM of (4.134) could be compared to the model presented in (4.92). Given that the absolute value of the elasticities used in this fixed input proportion example all equal 1, the expectation might be that a 10% reduction in the supply of input 1 should cause a 5% reduction in output. This outcome will occur if a 10% reduction in the supply of both inputs are simultaneously included in the ***b*** vector of the model. In that case, all endogenous variables are changed by (plus or minus) 5%.

For purposes of comparison, note that (4.134) indicates that the 10% decrease in the supply of input 1 decreases the use of input 1 by 4.5%. The EDM model in (4.95) predicted a decline of 6.7%. The equilibrium price of input 1 is estimated to increase by 5.5%, while the earlier model predicted a 3.3% increase.

The results also indicate that the use of input 2 decreases by 4.5%. This is identical to the reduction in the use of input 1 because of the assumption of no input substitution and the 4.5% reduction in output. In addition, the price of input 2 decreases by 4.5% because of the decreased demand for the input. The change in input demand is identical to the reduction in input price because of the assumed unitary supply elasticity.

From: Student@UEconomics.edu
To: Professor Watson
Date: Sunday, 18 Oct 2021 at 8:05 p.m.
Subject: Chapter 4

You gave an example of an EDM with vertical linkages and why accounting for homogenization of degree zero is important. The literature review discussed vertical linkages in the case of beef. Are there other EDMs outside of the beef industry?

Thanks! Sorry to bother you.
Shelly

From: Professor Watson
To: Student@UEconomics.edu
Date: Monday, 19 Oct 2021 at 7:20 a.m.
Re: Nonlinearities

Dear Class,

Shelly asks a good question. If you remember, the chapter noted that the assumption of perfect competition throughout the vertical chains would suggest that a single- versus multiple-stage model would yield similar results if the EDM were set up correctly as noted by the authors. However, it is possible to set up an EDM with an oligopolistic structure to model market power, as we will see in several weeks.

Historically, the beef supply chain has been characterized by four stages of production: cow-calf, backgrounding or stocker, feedlot, and processing. Data are readily available for all levels. The EDMs developed to model pork production have used three stages (feeder pig, finishing, and processing). An EDM that models the production of corn-based ethanol, distiller's grains, and high fructose corn syrup could be developed to analyze the impact of a change in agricultural regulations. For example, in 2021 a federal court ruled that the annual setting of the Renewable Fuels Standard for minimum ethanol production levels should consider the environmental impacts of corn production. That would also be an example of a two-stage vertical linkage because the environmental impact would likely be linked with nitrogen fertilizer and diesel fuel usage. Another example might be a labeling regulation regarding omega-3 vs omega-6 fatty acids on salmon to denote farm-fed vs wild salmon. Nutritionists recommend consuming omega-3 foods, which are found in higher concentrations in wild salmon. Fish that are farm-raised consume feed from different sources than wild salmon, which alters the composition of fatty acids. A second stage would include the production of fish feed and krill or herring. Data on the latter might be hard to find but I suspect, depending upon how the EDM was specified, you might not need it. My point is that beef has been widely studied because of the many policy issues impacting this sector and readily available data.

As an aside, one should not rely on their spell-checkers when communicating technical terms. "Homogenization" refers to a process, such as that used in fluid milk, that emulsifies liquids to prevent the separation of components. You meant to write "homogeneous." This is a common mistake!

All the best,
Dr. Watson

A Technological Change

A technological change that can easily be modeled within an EDM framework involves a technology shock that equally affects all aspects of an economic system, as discussed in Appendix 4B. Examples may include changes in information technology (e.g., computerization, block chain software, cloud computing) or improvements in educational outcomes. Such changes are most easily modeled if one assumes that the technological change is multiplicative in nature and affects production technologies for output and all inputs.

Consider the impact of a technology shock, α, that improves the productivity of all factors of production and the production function itself. In this case the initial production function and FOCs are of the form

$$q = \alpha f(\boldsymbol{x}) \tag{4.135}$$

$$p\alpha \boldsymbol{f_x} - w = 0. \tag{4.136}$$

Appendix 4B describes the process of converting these new functions into elasticity forms. Thus, the EDM model presented in (4.25)–(4.28) must be modified. Equations (4.25) and (4.28) remain the same, but (4B.31) and (4B.34) from Appendix 4B are substituted for (4.26) and (4.27), respectively, which results in

$$E(q^D) = \eta^D E(p^D) \tag{4.137}$$

$$E(p) = \sum_j K_j\, E(w_j) - E(\alpha) \tag{4.138}$$

$$E(x_i) = E(q) + \sum_j K_j\, \sigma_{ij} E(w_j) - E(\alpha),\ i = 1, 2, \dots, n \tag{4.139}$$

$$E(x_i) = \sum_j \varepsilon_{ij} E(w_j^S),\ i = 1, 2, \dots, n. \tag{4.140}$$

For ease of illustration, we continue to assume that input supply quantities are a function of only own-input prices such that $\varepsilon_{12} = \varepsilon_{21} = 0$. Thus, a one-output, two-input EDM can be written as

(4.141) $E(q) = \eta^D E(p) + E(\theta_1)$

(4.142) $E(p) = K_1 E(w_1) + K_2 E(w_2) - E(\alpha)$

(4.143) $E(x_1) = E(q) + K_1 \sigma_{11} E(w_1) + K_2 \sigma_{12} E(w_2) - E(\alpha)$

(4.144) $E(x_2) = E(q) + K_1 \sigma_{21} E(w_1) + K_2 \sigma_{22} E(w_2) - E(\alpha)$

(4.145) $E(x_1) = \varepsilon_1 E(w_1) + E(\theta_5)$

(4.146) $E(x_2) = \varepsilon_2 E(w_2) + E(\theta_6)$.

An improvement in technology is represented by a percentage change in α. Note that this change occurs simultaneously and equally in (4.142), (4.143), and (4.144). In addition, *E(α)* is substracted from the equations because improvements in technology reduce the quantity of inputs needed to produce a given level of output in (4.143) and (4.144) and, in a competitive market environment, reduce the price of the good or service (4.142).

The EDM model presented in (4.141)–(4.146) is operationalized by moving the endogenous variables to the left-hand side:

(4.147) $E(q) - \eta^D E(p) = E(\theta_1)$

(4.148) $E(p) - K_1 E(w_1) - K_2 E(w_2) = -E(\alpha)$

(4.149) $E(x_1) - E(q) - K_1 \sigma_{11} E(w_1) - K_2 \sigma_{12} E(w_2) = -E(\alpha)$

(4.150) $E(x_2) - E(q) - K_1 \sigma_{21} E(w_1) - K_2 \sigma_{22} E(w_2) = -E(\alpha)$

(4.151) $E(x_1) - \varepsilon_1 E(w_1) = E(\theta_5)$

(4.152) $E(x_2) - \varepsilon_2 E(w_2) = E(\theta_6)$.

Putting (4.147)–(4.152) into matrix notation yields

(4.153) $$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -K_1 & -K_2 \\ -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} \\ -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} \\ 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ -E(\alpha) \\ -E(\alpha) \\ -E(\alpha) \\ E(\theta_5) \\ E(\theta_6) \end{bmatrix}$$

To parameterize (4.153), we continue to use Gardner's (1988) example and assume that the own-price elasticity of demand, η^D, is −0.60; the own-price elasticities of input supply, (ε_1, ε_2), are 0.20 and 1.0, respectively; and the factor shares of x_1 (K_1) and x_2 (K_2) are 0.30 and 0.70, respectively. We assume the AES are $\sigma_{12} = \sigma_{21} = 1.0$, giving

$$\sigma_{11} = -\frac{K_2\sigma_{12}}{K_1} = -\frac{(0.70)(1.0)}{0.30} = -2.33 \text{ and}$$

$$\sigma_{22} = -\frac{K_1\sigma_{21}}{K_2} = -\frac{(0.30)(1.0)}{0.70} = -0.429.$$

In addition, we assume that an increase in technology causes the productivity of all factors of production to increase by 10%. Thus, (4.153) is parameterized as

$$(4.154)\quad \begin{bmatrix} 1 & 0.60 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.30 & -0.70 \\ -1 & 0 & 1 & 0 & 0.70 & -0.70 \\ -1 & 0 & 0 & 1 & -0.30 & 0.30 \\ 0 & 0 & 1 & 0 & -0.20 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1.0 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} 0.00 \\ -0.10 \\ -0.10 \\ -0.10 \\ 0.00 \\ 0.00 \end{bmatrix}$$

Solving (4.154) results in

$$(4.155)\quad \boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} 0.079 \\ -0.132 \\ -0.009 \\ -0.026 \\ -0.044 \\ -0.026 \end{bmatrix}.$$

The general increase in technology increases output, $E(q)$, by 7.9% and reduces output price, $E(p)$, by 13.2%. In addition, the use of inputs 1, $E(x_1)$, and 2, $E(x_2)$, declines by 0.9% and 2.6%, respectively. The price of input 1, $E(w_1)$, declines by 4.4%, and the price of input 2, $E(w_2)$, declines by 2.6%.

Interpreting the A^{-1} Matrix

The results obtained from a traditional comparative statics exercise and an EDM depend upon the properties of their associated inverse matrices. The examples in this chapter have presented results as changes in the endogenous variables, $\boldsymbol{y}$, but have not specifically discussed the useful information contained in the $\boldsymbol{A}^{-1}$ matrix. In this section, we examine several useful properties of that matrix.

Interpreting the A^{-1} Matrix for Behavioral Equation Shocks

The EDM inverse matrix contains linearly approximated endogenous variable responses to exogenous shocks as estimated by movements along hyperplanes tangent to the underlying nonlinear system, as shown in Figures 4.1 and 4.2. The EDM inverse matrix also accounts for simultaneous interactions among all variables included in the system. Because the dual EDM system was derived by performing elementary row operations on the primal system's total differential matrix, an EDM's inverse matrix can be used to obtain traditionally derivable comparative static results for an underlying primal system. These include signs on endogenous variable changes in response to either single or multiple exogenous shocks.

Table 4.1 presents the inverse matrix used to obtain the results in (4.45).[6] The *ij* elements in the matrix represent the marginal effects of a shock to an exogenous variable in column *j* on the endogenous variable in row *i*. For example, the value in the $E(q)$ row and 4.35 column in Table 4.1 indicates that a 1% change in $E(\theta_1)$ causes a 0.53% change in output. Therefore, for a 10% positive shock in $E(\theta_1)$, output increases by 5.3%. Note that this result is presented in (4.45). That is, if the first column of the inverse matrix in Table 4.1 is multiplied by $E(\theta_1) = 0.10$, we obtain the vector of endogenous responses presented in (4.45).

We can use the EDM's inverse matrix to examine the effect of other shocks as well. For example, column 4.39 of Table 4.1 presents the endogenous responses caused by a supply shock to input 1. The responses are obtained by multiplying the entries in column 4.39 by $E(\theta_5) = -0.10$. Similarly, the effect of a supply shock

Table 4.1. Interpretation of the A^{-1} Matrix

Endogenous Responses	Shocks to Behavioral Equations					
	4.35	**4.36**	**4.37**	**4.38**	**4.39**	**4.40**
$E(q)$	0.53	−0.32	−0.20	−0.28	0.20	0.28
$E(p)$	0.79	0.53	0.33	0.41	−0.33	−0.46
$E(x_1)$	0.22	−0.13	0.19	0.03	0.81	−0.03
$E(x_2)$	0.66	−0.39	0.07	0.59	−0.07	0.41
$E(w_1)$	1.10	−0.66	0.94	0.15	−0.94	−0.15
$E(w_2)$	0.66	−0.39	0.07	0.59	−0.07	−0.59

to input 2 can be obtained by multiplying column 4.40 by the relevant percentage. In this case, we see that larger output quantity and price responses in the first two rows to a given percentage shock in the supply of input 2 occur relative to the changes caused by the same percentage shock in the supply of input 1.

The inverse matrix presented in Table 4.1 makes it possible to estimate the endogenous responses when two or more shocks, $E(\theta_i)$, occur simultaneously.

6 Inverse matrices for all the book's numerical results are available in the associated Excel workbook.

For example, the results presented in (4.50) were obtained after setting $E(\theta_1) = -0.10$ and $E(\theta_5) = -0.10$. These are obtained by multiplying each entry of column 4.35 by −0.10 and summing the results with the values created by multiplying each entry of column 4.39 by −0.10.

Interpretation of the A^{-1} Matrix for Equilibrium Equation Shocks

When checking for HD0 in prices, we introduced an EDM that allowed for potential wedges or differences between demand and supply prices in (4.102)–(4.110). Table 4.2 presents the inverse of matrix of (4.123). While we defer a more complete discussion of price and quantity wedges until the next chapter, we note several points with respect to the inverse matrix of the fixed input proportion model (4.123). Equilibrium price wedges were introduced in (4.117), (4.118), and (4.119) as a means for assessing whether the EDM was HD0 in all prices. The equilibrium equations are associated with the values in the last three columns of Table 4.2.

When testing for HD0 in prices, we set $E(\theta_7) = -1.00$, $E(\theta_8) = 1.00$, and $E(\theta_9) = 1.00$. If the system is HD0 in output and all input prices, changes in all endogenous variables except $E(p^S)$, $E(w_1^D)$, and $E(w_2^D)$ should be 0. The inverse matrix presented in Table 4.2 provides an easy confirmation of homogeneity if each row entry in column 4.117 is equal to the sum of each row entry in columns 4.118–4.119 for all variables except $E(p^S)$, $E(w_1^D)$, and $E(w_2^D)$. For these three variables, the row entries in column 4.117 must equal the negative of the associated sum of the row entries in columns 4.118 and 4.119. An examination of Table 4.2 confirms that these conditions hold.

Table 4.2. Interpretation of the A^{-1} Matrix with Price Wedge Effects

Endogenous Responses	Shocks to Behavioral Equations						Shocks to Equilibrium Equations		
	4.111	4.112	4.113	4.114	4.115	4.116	4.117	4.118	4.119
$E(q)$	0.500	−0.500	−0.450	−0.050	0.450	0.050	−0.500	−0.450	−0.050
$E(p^D)$	0.500	0.500	0.450	0.050	−0.450	−0.050	0.500	0.450	0.050
$E(p^S)$	0.500	0.500	0.450	0.050	−0.450	−0.050	−0.500	0.450	0.050
$E(x_1)$	0.500	−0.500	0.550	−0.050	0.450	0.050	−0.500	−0.450	−0.050
$E(x_2)$	0.500	−0.500	−0.450	0.950	0.450	0.050	−0.500	−0.450	−0.050
$E(w_1^D)$	0.500	−0.500	0.550	−0.050	−0.550	0.050	−0.500	0.550	−0.050
$E(w_2^D)$	0.500	−0.500	−0.450	0.950	0.450	−0.950	−0.500	−0.450	0.950
$E(w_1^S)$	0.500	−0.500	0.550	−0.050	−0.550	0.050	−0.500	−0.450	−0.050
$E(w_2^S)$	0.500	−0.500	−0.450	0.950	0.450	−0.950	−0.500	−0.450	−0.050

Using the A^{-1} Matrix to Recover Implied Elasticities

The $\boldsymbol{A}^{-1}$ matrix can also be used to recover various total-response elasticities that are implied by an EDM. That is, parametrizing the $\boldsymbol{A}$ matrix requires the use of various elasticities and factor shares. However, some unspecified elasticity estimates may be useful to researchers and can be obtained from the $\boldsymbol{A}^{-1}$ matrix.

For example, consider the EDM presented in (4.111)–(4.119) and the parameters used to populate the $\boldsymbol{A}$ matrix presented in (4.44). The resulting $\boldsymbol{A}$ matrix is given by

$$\begin{bmatrix} 1 & 0.60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -0.30 & -0.70 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0.70 & -0.70 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & -0.30 & 0.30 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -0.20 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1.0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}. \tag{4.156}$$

Before proceeding, we again verify that the system presented in (4.156) is HD0 in output and input prices by examining its inverse matrix (see Table 4.3). Homogeneity can be checked by multiplying each row entry in column 4.117 by –1 and each row entry in columns 4.118 and 4.119 by 1. The sum of these products should be 0 for each row that represents percentage changes in output, $E(q)$; consumer price, $E(p^D)$; input quantities, $E(x_1)$ and $E(x_2)$; and input supply prices, $E(w_1^S)$ and $E(w_2^S)$.

Some elasticity estimates can also be gleaned from the equilibrium equation columns in Table 4.3. For example, consider (4.117), which places a price wedge between

Table 4.3. Elasticities and Homogeneity Using the $\boldsymbol{A}^{-1}$ Matrix with Price Wedge Effects

Endogenous Responses	Shocks to Behavioral Equations						Shocks to Equilibrium Equations		
	4.111	**4.112**	**4.113**	**4.114**	**4.115**	**4.116**	**4.117**	**4.118**	**4.119**
$E(q)$	0.526	−0.316	−0.197	−0.276	0.197	0.276	−0.316	−0.039	−0.276
$E(p^D)$	0.789	0.526	0.329	0.461	−0.329	−0.461	0.526	0.066	0.461
$E(p^S)$	0.789	0.526	0.329	0.461	−0.329	−0.461	−0.474	0.066	0.461
$E(x_1)$	0.219	−0.132	0.189	0.031	0.811	−0.031	−0.132	−0.162	0.031
$E(x_2)$	0.658	−0.395	0.066	0.592	−0.066	0.408	−0.395	0.013	−0.408
$E(w_1^D)$	1.096	−0.658	0.943	0.154	−0.943	−0.154	−0.658	0.189	0.154
$E(w_2^D)$	0.658	−0.395	0.066	0.592	−0.066	−0.592	−0.395	0.013	0.592
$E(w_1^S)$	1.096	−0.658	0.943	0.154	−0.943	−0.154	−0.658	−0.811	0.154
$E(w_2^S)$	0.658	−0.395	0.066	0.592	−0.066	−0.592	−0.395	0.013	−0.408

the price that consumers pay for a product and the price that producers receive for a product. The row entry for the percentage change in quantity consumed, $E(q)$, equals –0.316 and the row entry for the percentage change in consumer price, $E(p^D)$, equals 0.526. The own-price elasticity of demand, which is the percentage change in quantity divided by the percentage change in demand price, is calculated as

$$(4.157) \quad \eta^D = \frac{E(q)}{E(p^D)} = \frac{-0.316}{0.526} = -0.60,$$

which is the assumed own-price elasticity of demand used to parameterize the EDM.

Other nonspecified, but implied, total-response elasticities can be similarly obtained from Table 4.3. For example, the own-price elasticity of output supply was not needed to construct the EDM. Its implied value, however, can be extracted from Table 4.3. The own-price total-response elasticity of supply, or the percentage change in quantity divided by the percentage change in producer price, can be calculated using the row entries for $E(q)$ and $E(p^S)$ in column 4.117. That is, the own-price elasticity of supply ε^S is calculated as

$$(4.158) \quad \varepsilon^S = \frac{E(q)}{E(p^S)} = \frac{-0.316}{-0.474} = 0.67.$$

Note that the own-price elasticity of demand is more inelastic than the own-price elasticity of supply. Hence, if a sales or excise tax were placed on the output market, consumers would bear a larger incidence of the tax than producers (Mankiw, 2018).

Equation (4.118) places a price wedge between the demand price, $E(w_1^D)$, and the supply price, $E(w_1^S)$, of input 1. Although it was not specified in the construction of the EDM, the own-price elasticity of derived demand for input 1 can be obtained from Table 4.3. That is, the percentage change in the use of input 1, $E(x_1)$, divided by the percentage change in the demand price of input 1, $E(w_1^D)$, is given by

$$(4.159) \quad \eta_{x_1}^D = \frac{E(x_1)}{E(w_1^D)} = \frac{-0.162}{0.189} = -0.86.$$

Note that the assumed own-price elasticity of supply of input 1, 0.20, which is the percentage change in the use of input 1 divided by the percentage change in the producer price of input 1, is also represented in Table 4.3 as –0.162/–0.811 = 0.20.

Likewise, Table 4.3 can be used to estimate the own-price elasticity of demand for input 2. In this case, the column representing (4.119) is used because it places a price wedge between the demand price, $E(w_2^D)$, and the supply price, $E(w_2^S)$, of input 2. Therefore, the percentage change in the quantity of input 2, $E(x_2)$, divided by the percentage change in the demand price of input 2, $E(w_2^D)$, is given by

(4.160) $$\eta_{x_2}^D = \frac{E(x_2)}{E(w_2^D)} = \frac{-0.408}{0.592} = -0.69.$$

For this EDM, the own-price elasticity of demand for input 2 is more inelastic, –0.69, than the own-price elasticity of demand for input 1, –0.86.

We conclude this discussion on the use of $\boldsymbol{A}^{-1}$ matrices by noting that they can be used to derive numerous other response metrics. For example, the system's total-response cross-price elasticities of derived demand can also be recovered from rows $E(x_1)$ and $E(w_2^D)$ in column 4.119 and rows $E(x_2)$ and $E(w_1^D)$ in column 4.118 to yield

(4.161) $$\eta_{x_1,w_2}^D = \frac{E(x_1)}{E(w_2^D)} = \frac{0.031}{0.592} = 0.05$$

and

(4.162) $$\eta_{x_2,w_1}^D = \frac{E(x_2)}{E(w_1^D)} = \frac{0.013}{0.189} = 0.07.$$

The positive values for these cross-price elasticities of derived input demand indicate that the two inputs are substitutes in the production process.

Summary

Several approaches have been used to develop EDMs. Some methodologies and studies suffer from a fundamental flaw in that they do not explicitly impose HD0 in input and output prices. This oversight creates bias in estimates of price and quantity equilibria that result from exogenous economic or policy shocks as well as for changes in surplus.

Our development of a theoretically consistent EDM follows Allen (1938) by converting the primal problem of profit maximization into its dual structure — although Allen did not term this a "dual" structure. The conversion allows the use of elasticity estimates and factor shares to parameterize an EDM. Others have developed theoretically consistent EDMs using a cost function and applying Shephard's Lemma — an approach that requires several assumptions (Zhang, 2021). In both cases, researchers should always check EDMs to be certain that their models are consistent with homogeneity conditions. Such a check provides a means for identifying modeling mistakes and produces results that are consistent with economic theory.

»Chapter Four, Appendix A

HOMOGENEITY OF DEGREE 1 PRODUCTION FUNCTIONS AND THE DEVELOPMENT OF A DUAL EDM SYSTEM

At least two methods can be used to derive theoretically consistent EDM models. In this appendix, we follow the original approach developed by Allen (1938). Some authors have developed EDMs by applying Shephard's Lemma to cost functions and using several additional assumptions (e.g., Mullen, Wohlgenant, and Farris, 1988; Zhang, 2021). We have chosen to use Allen's primal-based approach as this allows practitioners to identify and model potential shocks in the underlying primal/production space when deriving appropriate EDM modifications.

If a long-run production function, $q = f(x)$, is HD1 in inputs, then the following expressions hold:

(4A.1) $$q = f(t\boldsymbol{x}) = tf(\boldsymbol{x})$$

(4A.2) $$\boldsymbol{H} = \left[\frac{\partial^2 f}{\partial x_i \partial x_i}\right] = \left[f_{ij}\right]$$

(4A.3) $$\sum_j f_j(\boldsymbol{x})x_j = \boldsymbol{f_x x} = q$$

(4A.4) $$\sum_j f_{i,j}(\boldsymbol{x})x_j = \sum_i f_{i,j}(\boldsymbol{x})x_i = 0 \Rightarrow H(\boldsymbol{x})\boldsymbol{x} = 0;\ \boldsymbol{x}H(\boldsymbol{x}) = 0;\ |\boldsymbol{H}| = 0.$$

A consequence of (4A.1)–(4A.4) is that the unconstrained competitive FOCs given by

(4A.5) $$p\boldsymbol{f_x} - \boldsymbol{w} = 0$$

have an infinite number of solutions. While the FOCs do not have a unique solution, we assume that they will be satisfied. In this case (4A.1)–(4A.5) imply 0 profits for the competitive firm and industry, which can be shown as

(4A.6) $$p\boldsymbol{f_x}x = \boldsymbol{wx} \Leftrightarrow pq = \boldsymbol{wx}.$$

Gardner (1988) suggests simultaneously treating output, q, in (4A.1) as exogenous and output price, p, in (4A.5) as endogenous as a means for arriving at a unique solution to the simultaneous augmented first order system:

(4A.7) $a: q = f(\boldsymbol{x})$

$b: p\boldsymbol{f_x} - \boldsymbol{w} = 0.$

Using (4A.7) can be intuitively justified by considering it as part of a larger simultaneous system of equations that also endogenously determines q:

(4A.8) $a: \; q = q^D(p)$

$b: \; q = f(\boldsymbol{x})$

$c: \; p\boldsymbol{f_x} - \boldsymbol{w} = 0$

$d: \; \boldsymbol{x} = \boldsymbol{x}^S(\boldsymbol{w}),$

where q is output, p is output price, $\boldsymbol{x}$ is an n-vector of inputs, $\boldsymbol{w}$ is a vector of input prices, q^D is a demand equation, $\boldsymbol{x}^S$ and is a set of input supply functions. This system of equations consists of $2n + 2$ equations and $2n + 2$ endogenous variables (q, p, $\boldsymbol{x}$, and $\boldsymbol{w}$). Although the FOCs of (4A.8) do not have a unique solution when examined in isolation, the implicit function theorem (IFT) can be used to show that an implicit local solution to the system exists. For example, begin by taking the total differential of (4A.8):

(4A.9) $a: \; dq = \left(\frac{\partial q^D}{\partial p}\right) dp$

$b: \; dq = \boldsymbol{f_x}\, \boldsymbol{dx}$

$c: \; \boldsymbol{f_x} dp + p\boldsymbol{f_{xx'}}\, \boldsymbol{dx} - \boldsymbol{I}\, \boldsymbol{dw} = 0$

$d: \; \boldsymbol{dx} = \left(\frac{\partial \boldsymbol{x}^S}{\partial \boldsymbol{x}}\right) \boldsymbol{dw}.$

After matrix row operations and some algebraic manipulation, this system of equations can be shown to be equivalent to the following EDM system:

(4A.10) $a{:}\ E(q) = \eta^D E(p)$

$b{:}\ E(p) = \sum_j K_j E(w_j)$

$c{:}\ E(x_i) = E(q) + \sum_j K_j \sigma_{ij} E(w_j), i = 1, 2, \ldots, n$

$d{:}\ E(x_i) = \sum_j K_j \varepsilon_{ij} E(w_j),\ i = 1, 2, \ldots, n.$

Note that (4A.10a) and (4A.10d) are derived using procedures described in Chapter 4. Procedures for deriving equations (4A.10b) and (4A.10c) are described in the following section.

Developing the EDM for the Production Sector

We expand (4A.9b) and (4A.9c) to obtain

(4A.11)
$$\begin{bmatrix} 0 & f_1 & f_2 & \cdot & f_n \\ f_1 & pf_{11} & pf_{12} & \cdot & pf_{1n} \\ f_2 & pf_{21} & pf_{22} & \cdot & pf_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & pf_{n1} & pf_{n2} & \cdot & pf_{nn} \end{bmatrix} \begin{bmatrix} dp \\ dx_1 \\ dx_2 \\ \cdot \\ \cdot \\ \cdot \\ dx_n \end{bmatrix} = \begin{bmatrix} dq \\ dw_1 \\ dw_2 \\ \cdot \\ \cdot \\ \cdot \\ dw_n \end{bmatrix}.$$

If the cost minimization problem discussed below has a solution, the bordered matrix in (4A.11) is nonsingular and, by the IFT, implicit solutions

(4A.12) $p^* = p^*(q, \boldsymbol{w})$

and

(4A.13) $\boldsymbol{x}^* = \boldsymbol{x}^*(q, \boldsymbol{w})$

exist which imply the following dual differential system of equations:

(4A.14) $dp^* = \left(\frac{\partial p^*}{\partial q}\right) dq + \sum_j \left(\frac{\partial p^*}{\partial w_j}\right) dw_j$

(4A.15) $dx_i^* = \left(\frac{\partial x_i^*}{\partial q}\right) dq + \sum_j \left(\frac{\partial x_i^*}{\partial w_j}\right) dw_j.$

Because this system of dual equations reduces the complexity of the mathematical computations, we use (4A.11) and Cramer's Rule to identify expressions for $\frac{\partial p^*}{\partial q}, \frac{\partial p^*}{\partial w_j}, \frac{\partial x_i^*}{\partial q}$ and $\frac{\partial x_i^*}{\partial w_j}$. The results are used to construct equations (4A.10b) and (4A.10c) in the EDM. To identify expressions for $\frac{\partial p^*}{\partial q}, \frac{\partial p^*}{\partial w_j}, \frac{\partial x_i^*}{\partial q}$ and $\frac{\partial x_i^*}{\partial w_j}$, we use results motivated by Allen (1938). Let

(4A.16) $$\boldsymbol{F} = \begin{bmatrix} 0 & f_1 & \cdot & f_j & \cdot & f_n \\ f_1 & f_{11} & \cdot & f_{1j} & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_i & f_{i1} & \cdot & f_{ij} & \cdot & f_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & f_{n1} & \cdot & f_n & \cdot & f_{nn} \end{bmatrix} = \begin{bmatrix} 0 & \boldsymbol{f}_{\boldsymbol{x}}' \\ \boldsymbol{f}_{\boldsymbol{x}} & \boldsymbol{H} \end{bmatrix}.$$

In the following, we denote the borders as row or column 0 and refer to the remaining rows and columns using the indexes 1, … , n which results in the following definitions:

- F_0 ≡ the cofactor of the 0 element in matrix $\boldsymbol{F}$.
- $F_{0,ij}$ ≡ the cofactor of the ij^{th} element in $\boldsymbol{F}$ after the borders have first been deleted.
- $\boldsymbol{F}_{i,j}$ ≡ the matrix remaining after the i^{th} row and j^{th} column in $\boldsymbol{F}$ have been deleted.

Given these definitions, the following results are obtained:

(4A.17) $$F_0 = 0$$

(4A.18) $$F_{0,ij} = (-1)^{i+j+1} \frac{x_i x_j |\boldsymbol{F}|}{q^2}$$

(4A.19) $$\frac{|\boldsymbol{F}_{ij}|}{|\boldsymbol{F}|} = (-1)^{i+j} \frac{\sigma_{ij} x_i x_j}{q} = (-1)^{i+j} \frac{K_j \sigma_{ij} x_i}{f_j},$$

where $\sigma_{i,j}$ is the AES and

$K_j = \frac{w_j x_j}{\boldsymbol{wx}}$ is input j's factor share. Recall that $\sum_j K_j = 1$ and $\sum_j K_j \sigma_{ij} = 0$.

Before proceeding, we prove results (4A.17), (4A.18), and (4A.19).

Proof of (4A.17)

$F_0 = |\boldsymbol{H}| = 0$ from (4A.4).

Q.E.D.

Proof of (4A.18)

(4A.20) $$|\boldsymbol{F}| = \frac{1}{x_i}\begin{vmatrix} 0 & f_1 & \cdot & f_j & \cdot & f_n \\ f_1 & f_{11} & \cdot & f_{1j} & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_i x_i & f_{i1}x_i & \cdot & f_{ij}x_i & \cdot & f_{in}x_i \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & f_{n1} & \cdot & f_{nj} & \cdot & f_{nn} \end{vmatrix}.$$

Multiplying all rows $h > 0$ and $h \neq i$ by x_h and adding the results to row i gives, via (4A.3) and (4A.4),

(4A.21) $$|\boldsymbol{F}| = \frac{1}{x_i}\begin{vmatrix} 0 & f_1 & \cdot & f_j & \cdot & f_n \\ f_1 & f_{11} & \cdot & f_{1j} & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ q & 0 & \cdot & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & f_{n1} & \cdot & f_{nj} & \cdot & f_{nn} \end{vmatrix} = (-1)^{i+2}\frac{q}{x_i}\begin{vmatrix} f_1 & \cdot & f_j & \cdot & f_n \\ f_{11} & \cdot & f_{1j} & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \sim_i & \sim_i & \sim_i & \sim_i & \sim_i \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{n1} & \cdot & f_{nj} & \cdot & f_{nn} \end{vmatrix} =$$

$$(-1)^{i}\frac{q}{x_i}\begin{vmatrix} f_1 & \cdot & f_j & \cdot & f_n \\ f_{11} & \cdot & f_{1j} & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \sim_i & \sim_i & \sim_i & \sim_i & \sim_i \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{n1} & \cdot & f_{nj} & \cdot & f_{nn} \end{vmatrix},$$

where the notation $\sim_i$ or $\sim_j$ denotes that row *i* (or column *j*) has been deleted.

Multiplying column *j* by x_j gives:

(4A.22) $$|\boldsymbol{F}| = (-1)^{i}\frac{q}{x_i x_j}\begin{vmatrix} f_1 & \cdot & f_j x_j & \cdot & f_n \\ f_{11} & \cdot & f_{1j}x_j & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \sim_i & \sim_i & \sim_i & \sim_i & \sim_i \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{n1} & \cdot & f_{nj}x_j & \cdot & f_{nn} \end{vmatrix}.$$

Multiplying all columns $h > 0$ and $h \neq j$ by x_h, adding the results to column *j*, and using (4A.3) and (4A.4) gives:

(4A.23) $$|\boldsymbol{F}| = (-1)^{i}\frac{q}{x_i x_j}\begin{vmatrix} f_1 & \cdot & q & \cdot & f_n \\ f_{11} & \cdot & 0 & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \sim_i & \sim_i & \sim_i & \sim_i & \sim_i \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{n1} & \cdot & 0 & \cdot & f_{nn} \end{vmatrix} = (-1)^{i}(-1)^{j+1}\left(\frac{q^2}{x_i x_j}\right)F_{0,ij},$$

which reduces to

$$F_{0,ij} = (-1)^{i+j+1}\left(\frac{x_i x_j}{q^2}\right)|\boldsymbol{F}|. \tag{4A.24}$$

Q.E.D.

Proof of (4A.19)

To prove (4A.19) we use the following cost-minimization Lagrangian:

$$\mathcal{L} = \boldsymbol{wx} + \lambda[q - f(\boldsymbol{x})] \tag{4A.25}$$

with FOCs

$$\begin{aligned} &a\text{: } q - f(\boldsymbol{x}) = 0 \\ &b\text{: } \boldsymbol{w} - \lambda\, \boldsymbol{f_x} = 0. \end{aligned} \tag{4A.26}$$

Totally differentiating and rearranging gives

$$\begin{bmatrix} 0 & f_1 & \cdot & f_j & \cdot & f_n \\ f_1 & \lambda f_{11} & \cdot & \lambda f_{1j} & \cdot & \lambda f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_i & \lambda f_{i1} & \cdot & \lambda f_{ij} & \cdot & \lambda f_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & \lambda f_{n1} & \cdot & \lambda f_{nj} & \cdot & \lambda f_{nn} \end{bmatrix} \begin{bmatrix} d\lambda \\ dx_1^c \\ \cdot \\ dx_j^c \\ \cdot \\ dx_n^c \end{bmatrix} = \begin{bmatrix} dq \\ dw_1 \\ \cdot \\ dw_j \\ \cdot \\ dw_n \end{bmatrix}. \tag{4A.27}$$

In the following, we denote the bordered Hessian in (4A.27) as $\boldsymbol{L}^C$ and the matrix after deleting the i^{th} row and j^{th} column of $\boldsymbol{L}^C$ as $\boldsymbol{L}^C_{ij}$. Using Cramer's Rule we obtain

$$\frac{\partial x_i^c}{\partial w_j} = \frac{(-1)^{i+j+2}\left|\boldsymbol{L}^c_{ij}\right|}{|\boldsymbol{L}^c|} = \frac{(-1)^{i+j}\lambda^{n-2}}{\lambda^{n-1}}\frac{|\boldsymbol{F}_{ij}|}{|\boldsymbol{F}|} = \frac{(-1)^{i+j}|\boldsymbol{F}_{ij}|}{\lambda|\boldsymbol{F}|}. \tag{4A.28}$$

Using results from Silberberg (1990, pp. 316–317), Allen (1938, p. 504), and expression (4A.28), the AES can be written as

$$\begin{aligned} \sigma_{ij} &= \left(\frac{\partial x_i^c}{\partial w_j}\frac{w_j}{x_i}\right)\frac{1}{K_j} = \left(\frac{\partial x_i^c}{\partial w_j}\frac{w_j}{x_i}\right)\frac{\boldsymbol{wx}}{w_j x_j} \\ &= \frac{(-1)^{i+j}|\boldsymbol{F}_{ij}|}{\lambda|\boldsymbol{F}|}\frac{\lambda \boldsymbol{f_x x}}{x_i x_j} = (-1)^{i+j}\frac{\boldsymbol{f_x x}}{x_i x_j}\frac{|\boldsymbol{F}_{ij}|}{|\boldsymbol{F}|}. \end{aligned} \tag{4A.29}$$

If the production function is HD1 in prices (i.e., $\boldsymbol{f_x x} = q$), then (4A.29) reduces to

(4A.30) $\sigma_{ij} = (-1)^{i+j} \frac{q}{x_i x_j} \frac{|\boldsymbol{F}_{ij}|}{|\boldsymbol{F}|}$

or

(4A.31) $\frac{|\boldsymbol{F}_{ij}|}{|\boldsymbol{F}|} = (-1)^{i+j} \frac{\sigma_{ij} x_i x_j}{q}$.

Note also that Silberberg's result, $\sigma_j = \left(\frac{\partial x_i^c}{\partial w_j}\frac{w_j}{x_i}\right)\frac{1}{K_j}$, and expressions (4A.26) and (4A.28) give

(4A.32) $K_j\sigma_{ij} = \frac{\partial x_i^c}{\partial w_j}\frac{w_j}{x_i} = (-1)^{i+j}\frac{|\boldsymbol{F}_{ij}|}{\lambda|\boldsymbol{F}|}\frac{\lambda f_j}{x_i} = (-1)^{i+j}\frac{f_j}{x_i}\frac{|\boldsymbol{F}_{ij}|}{|\boldsymbol{F}|}$

or

(4A.33) $\frac{|\boldsymbol{F}_{ij}|}{|\boldsymbol{F}|} = (-1)^{i+j}\frac{K_j\sigma_{ij}x_i}{f_j}$.

Q.E.D.

Deriving Solutions for the Endogenous Variables

Given (4A.17), (4A.18), and (4A.19), we now derive expressions for

$$\frac{\partial p^*}{\partial q}, \frac{\partial p^*}{\partial w_j}, \frac{\partial x_i^*}{\partial q} \text{ and } \frac{\partial x_i^*}{\partial w_j}.$$

To motivate this discussion, we restate the system of equations in (4A.11) as

(4A.34) $$\begin{bmatrix} 0 & f_1 & \cdot & f_j & \cdot & f_n \\ f_1 & pf_{11} & \cdot & pf_{1j} & \cdot & pf_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_i & pf_{i1} & \cdot & pf_{ij} & \cdot & pf_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & pf_{n1} & \cdot & pf_{nj} & \cdot & pf_{nn} \end{bmatrix}\begin{bmatrix} dp \\ dx_1 \\ \cdot \\ dx_j \\ \cdot \\ dx_n \end{bmatrix} = \begin{bmatrix} dq \\ dw_1 \\ \cdot \\ dw_j \\ \cdot \\ dw_n \end{bmatrix}.$$

Denoting the bordered matrix in (4A.34) as $\boldsymbol{H}_p$ and using Cramer's Rule, we obtain

(4A.35) $$\frac{\partial p^*}{\partial q} = \frac{\begin{vmatrix} 1 & f_1 & \cdot & f_j & \cdot & f_n \\ 0 & pf_{11} & \cdot & pf_{1j} & \cdot & pf_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & pf_{i1} & \cdot & pf_{ij} & \cdot & pf_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & pf_{n1} & \cdot & pf_{nj} & \cdot & pf_{nn} \end{vmatrix}}{|\boldsymbol{H}_p|} = \frac{p^n\begin{vmatrix} 1 & f_{11} & \cdot & f_{1j} & \cdot & f_{1n} \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & f_{i1} & \cdot & f_{ij} & \cdot & f_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & f_{n1} & \cdot & f_{nj} & \cdot & f_{nn} \end{vmatrix}}{p^{n-1}|\boldsymbol{F}|} = \frac{p\boldsymbol{F}_0}{|\boldsymbol{F}|} = 0$$

or

(4A.36) $$\frac{\partial p^*}{\partial q} = 0.$$

The symmetry and nonsingularity of $\boldsymbol{H}_p$ imply that $\frac{\partial p^*}{\partial w_j} = \frac{\partial x_i^*}{\partial q}$. From (4A.34), we obtain

(4A.37) $$\frac{\partial x_j^*}{\partial q} = \frac{\begin{vmatrix} 0 & f_1 & \cdot & 1 & \cdot & f_n \\ f_1 & pf_{11} & \cdot & 0 & \cdot & pf_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_j & pf_{j1} & \cdot & 0 & \cdot & pf_{jn} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & pf_{n1} & \cdot & 0 & \cdot & pf_{nn} \end{vmatrix}}{|\boldsymbol{H}_p|} = \frac{(-1)^{j+2}\frac{p^{n-1}}{x_j}\begin{vmatrix} f_1 & f_{11} & \cdot & \sim j & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ f_j x_j & f_{j1}x_j & \cdot & \sim j & \cdot & f_{jn}x_j \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ f_n & f_{n1} & \cdot & \sim j & \cdot & f_{nn} \end{vmatrix}}{p^{n-1}|\boldsymbol{F}|}.$$

Adding x_i times row i to row j for $i \neq j$ and using results (4A.3) and (4A.4) gives

(4A.38) $$\frac{\partial x_j^*}{\partial q} = \frac{(-1)^{j+2}\frac{p^{n-1}}{x_j}\begin{vmatrix} f_1 & f_{11} & \cdot & \sim j & \cdot & f_{1n} \\ f_2 & f_{21} & \cdot & \sim j & \cdot & f_{2n} \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ q & 0 & \cdot & \sim j & \cdot & 0 \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ f_{n1} & f_{n1} & \cdot & \sim j & \cdot & f_{nn} \end{vmatrix}}{p^{n-1}|\boldsymbol{F}|}$$

$$= (-1)^{j+2}(-1)^{j+1}\frac{qF_{0,jj}}{x_j|\boldsymbol{F}|} = (-1)^{2j+3}\frac{qF_{0,jj}}{x_j|\boldsymbol{F}|}.$$

Substituting for $F_{0,jj}$ yields

(4A.39) $$\frac{\partial x_j^*}{\partial q} = \frac{q}{x_j}\frac{x_j x_j}{q^2}\frac{|\boldsymbol{F}|}{|\boldsymbol{F}|} = \frac{x_j}{q},$$

which results in

(4A.40) $$\frac{\partial x_j^*}{\partial q} = \frac{x_j}{q}$$

and, by symmetry,

(4A.41) $$\frac{\partial p^*}{\partial w_j} = \frac{x_j}{q}.$$

From (4A.34),

$$\text{(4A.42)} \quad \frac{\partial x_i^*}{\partial w_j} = \frac{\begin{vmatrix} 0 & f_1 & \cdot & 0 & \cdot & f_n \\ f_1 & pf_{11} & \cdot & 0 & \cdot & pf_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_j & pf_{j1} & \cdot & 1 & \cdot & pf_{jn} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & pf_{n1} & \cdot & 0 & \cdot & pf_{nn} \end{vmatrix}}{p^{n-1}\,|\boldsymbol{F}|}$$

$$= \frac{(-1)^{i+j+2}\begin{vmatrix} 0 & f_1 & f_2 & \sim i & \cdot & f_n \\ f_1 & pf_{11} & pf_{12} & \sim i & \cdot & pf_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sim j & \sim j & \sim j & \sim ji & \cdot & \sim j \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & pf_{n1} & pf_{n2} & \sim i & \cdot & pf_{nn} \end{vmatrix}}{p^{n-1}|\boldsymbol{F}|} = (-1)^{i+j}\,\frac{p^{n-2}|\boldsymbol{F}_{ij}|}{p^{n-1}|\boldsymbol{F}|}.$$

Substituting using (4A.33) and (4A.5) gives

$$\text{(4A.43)} \quad \frac{\partial x_i^*}{\partial w_j} = (-1)^{i+j}(-1)^{i+j}\,\frac{K_j\sigma_{ij}x_i}{pf_1}$$

or

$$\text{(4A.44)} \quad \frac{\partial x_i^*}{\partial w_j} = \frac{K_j\sigma_{ij}x_i}{w_j}.$$

Substituting (4A.36), (4A.40), (4A.41), and (4A.44) into (4A.14) and (4A.15) gives

$$\text{(4A.45)} \quad dp = \sum_j \frac{x_j}{q}\,dw_j$$

and

$$\text{(4A.46)} \quad dx_i = \frac{x_i}{q}\,dq + \sum_j \frac{K_j\sigma_{ij}x_i}{w_j}\,dw_j.$$

The results in (4A.45) and (4A.46) can be converted to an EDM as

$$\text{(4A.47)} \quad \frac{dp}{p} = \sum_j \frac{w_jx_j}{pq}\frac{dw_j}{w_j} = \sum_j \frac{w_jx_j}{\boldsymbol{wx}}\frac{dw_j}{w_j} = \sum_j K_j\,\frac{dw_j}{w_j}$$

or

$$\text{(4A.48)} \quad E(p) = \sum_j K_j E\left(w_j\right)$$

and

$$\text{(4A.49)} \quad \frac{dx}{x_i} = \frac{x_i}{x_i\,q}\,dq + \sum_j \frac{K_j\sigma_{ij}x_i}{x_iw_j}\,dw_j = \frac{dq}{q} + \sum_j K_j\sigma_{ij}\,\frac{dw_j}{w_j}$$

or

(4A.50) $$E(x_i) = E(q) + \sum_j K_j \sigma_{ij} E(w_j).$$

The result of this exercise is that the EDM approximation of the system in (4A.7) can be written as

(4A.51) $$q = f(\boldsymbol{x}) \approx E(p) = \sum_j K_j E(w_j)$$

$$p\boldsymbol{f_x} - \boldsymbol{w} = 0 \approx E(x_i) = E(q) + \sum_j K_j \sigma_{ij} E(w_j),\ i = 1, 2, \ldots, n$$

Consequently, the effects of exogenous shocks on a market that includes consumers, producers, and input suppliers can be approximated using an EDM of the form:

(4A.52) $$a\colon\ q = q^D(p) \approx E(q) = \eta^D E(p)$$

$$b\colon\ q = f(\boldsymbol{x}) \approx E(p) = \sum_j K_j E(w_j)$$

$$c\colon\ p\boldsymbol{f_x} - \boldsymbol{w} = 0 \approx E(x_i) = E(q) + \sum_j K_j \sigma_{ij} E(w_j) \quad i = 1, 2, \ldots, n$$

$$d\colon\ \boldsymbol{x} = \boldsymbol{x}^S(\boldsymbol{w}) \approx E(x_i) = \sum_j K_j \varepsilon_{ij} E(w_j) \quad i = 1, 2, \ldots, n.$$

These results are used to motivate the development of EDMs discussed in this book. We conclude this appendix by recommending that readers familiarize themselves with both the Allen-based and the more direct cost function approach, which uses Shephard's Lemma. For many users, the dual cost function-based approach may involve substantially less effort. However, we have found the Allen-based approach to be useful when considering exogenous shocks that differentially affect the productivity of inputs.

»Chapter Four, Appendix B

MULTIPLICATIVE TECHNOLOGY SHOCKS

Assume that a long-run production function, $q = f(\boldsymbol{x})$, is HD1 in inputs. Consider the impact of a technology shock α that improves the productivity of all factors of production and the production function itself. Such a shock could be the result of changes in information technologies or improved educational outcomes. In this case the system becomes

(4B.1) $$q = \alpha f(\boldsymbol{x})$$

(4B.2) $$p\alpha \boldsymbol{f_x} - w = 0.$$

The total differential of the system yields

(4B.3) $$dq = q d\alpha + \alpha \boldsymbol{f_x}\, \boldsymbol{dx}$$

(4B.4) $$\alpha \boldsymbol{f_x}\, d\alpha + \alpha \boldsymbol{f_x}\, dp + p\alpha\, \boldsymbol{f_{xx'}} - \boldsymbol{I}\, \boldsymbol{dw} = 0$$

or

(4B.5) $$\begin{bmatrix} 0 & \alpha f_1 & \cdot & \alpha f_n \\ \alpha f_1 & pf\alpha_{11} & \cdot & p\alpha f_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \alpha f_n & p\alpha f_{n1} & \cdot & p\alpha f_{nn} \end{bmatrix} \begin{bmatrix} dp^* \\ dx_1^* \\ \cdot \\ \cdot \\ \cdot \\ dx_n^* \end{bmatrix} = \begin{bmatrix} dq \\ dw_1 \\ \cdot \\ \cdot \\ \cdot \\ dw_n \end{bmatrix} - \begin{bmatrix} q \\ pf_1 \\ \cdot \\ \cdot \\ \cdot \\ pf_n \end{bmatrix} d\alpha.$$

Using the implicit function theorem (IFT), we obtain

(4B.6) $$p^* = p^*(q, \boldsymbol{w}, \alpha)$$

$$\boldsymbol{x}^* = \boldsymbol{x}^*(q, \boldsymbol{w}, \alpha),$$

resulting in

(4B.7) $$dp^* = \left(\frac{\partial p^*}{\partial q}\right) dq + \sum_j \left(\frac{\partial p^*}{\partial w_j}\right) dw_j + \left(\frac{\partial p^*}{\partial \alpha}\right) d\alpha$$

(4B.8) $$dx_i^* = \left(\frac{\partial x_i^*}{\partial q}\right) dq + \sum_j \left(\frac{\partial x_i^*}{\partial w_j}\right) dw_j + \left(\frac{\partial x_i^*}{\partial \alpha}\right), i = 1, 2, \dots, n.$$

We begin by examining the properties of

(4B.9) $$\widetilde{\boldsymbol{F}} = \begin{bmatrix} 0 & \alpha f_1 & \cdot & \alpha f_n \\ \alpha f_1 & \alpha f_{11} & \cdot & \alpha f_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \alpha f_1 & \alpha f_{n1} & \cdot & \alpha f_{nn} \end{bmatrix},$$

where

$\widetilde{F}_0 \equiv$ cofactor of element 0.

Let,

$\widetilde{F}_{0,ij} \equiv$ cofactor of the *ij* element after borders are deleted

$\widetilde{\boldsymbol{F}}_{ij} \equiv$ matrix remaining after element *ij* is deleted while retaining the borders.

Then,

(4B.10) $$|\widetilde{\boldsymbol{F}}| = \alpha^{n+1}|\boldsymbol{F}| \Rightarrow |\boldsymbol{F}| = \alpha^{-(n+1)}|\widetilde{\boldsymbol{F}}| = \frac{|\widetilde{\boldsymbol{F}}|}{\alpha^{n+1}}$$

(4B.11) $$\widetilde{\boldsymbol{F}}_{0,ij} = \alpha^n \boldsymbol{F}_{0,ij} = \alpha^n(-1)^{i+j+1}\frac{x_i x_j}{q^2}|\boldsymbol{F}| = (-1)^{i+j+1}\frac{x_i x_j \alpha^n}{q^2\alpha^{n+1}}|\widetilde{\boldsymbol{F}}| \Rightarrow$$

$$\widetilde{\boldsymbol{F}}_{0,ij} = (-1)^{i+j+1}\frac{x_i x_j}{\alpha q^2}|\widetilde{\boldsymbol{F}}|$$

(4B.12) $$\frac{|\widetilde{\boldsymbol{F}}_{ij}|}{|\widetilde{\boldsymbol{F}}|} = \frac{\alpha^n}{\alpha^{n+1}}\frac{|\boldsymbol{F}_{ij}|}{|\boldsymbol{F}|} = (-1)^{i+j}\frac{\sigma_{ij}x_i x_j}{\alpha q} = (-1)^{i+j}\frac{K_j}{\alpha}\frac{\sigma_{ij}x_i}{f_j}$$

(4B.13) $$\frac{\partial p^*}{\partial q} = \frac{\begin{vmatrix} 1 & \alpha f_1 & \cdot & \alpha f_j & \cdot & \alpha f_n \\ 0 & \alpha p f_{11} & \cdot & \alpha p f_{1j} & \cdot & \alpha p f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \alpha p f_{i1} & \cdot & \alpha p f_{ij} & \cdot & \alpha p f_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \alpha p f_{n1} & \cdot & \alpha p f_{nj} & \cdot & \alpha p f_{nn} \end{vmatrix}}{|\widetilde{\boldsymbol{H}}_p|} = \frac{\alpha^n p^n \begin{vmatrix} f_{11} & \cdot & f_{1j} & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{i1} & \cdot & f_{ij} & \cdot & f_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{n1} & \cdot & f_{nj} & \cdot & f_{nn} \end{vmatrix}}{\alpha^{n+1}p^{n-1}|\boldsymbol{F}|} = 0$$

or

(4B.14) $$\frac{\partial p^*}{\partial q} = 0.$$

By symmetry, $\frac{\partial p^*}{\partial w_j} = \frac{\partial x_j^*}{\partial q}$, with

(4B.15) $$\frac{\partial x_j^*}{\partial q} = \frac{\begin{vmatrix} 0 & \alpha f_1 & \cdot & 1 & \cdot & \alpha f_n \\ \alpha f_1 & \alpha p f_{11} & \cdot & 0 & \cdot & \alpha p f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \alpha f_j & \alpha p f_{j1} & \cdot & 0 & \cdot & \alpha p f_{jn} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \alpha f_n & \alpha p f_{n1} & \cdot & 0 & \cdot & \alpha p f_{nn} \end{vmatrix}}{|H_p|} = \frac{(-1)^{j+2}\begin{vmatrix} \alpha f_1 & \alpha p f_{11} & \cdot & \sim j & \cdot & \alpha p f_{1n} \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ \alpha f_j & \alpha p f_{j1} & \cdot & \sim j & \cdot & \alpha p f_{jn} \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ \alpha f_n & \alpha p f_{n1} & \cdot & \sim j & \cdot & \alpha p f_{nn} \end{vmatrix}}{\alpha^{n+1} p^{n-1} |\boldsymbol{F}|}$$

(4B.16) $$\frac{(-1)^j \alpha^n p^{n-1}\begin{vmatrix} f_1 & f_{11} & \cdot & \sim j & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ f_j & f_{j1} & \cdot & \sim j & \cdot & f_{jn} \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ f_n & f_{n1} & \cdot & \sim j & \cdot & f_{nn} \end{vmatrix}}{\alpha^{n+1} p^{n-1} |\boldsymbol{F}|} = \frac{\frac{(-1)^j}{x_j}\begin{vmatrix} f_1 & f_{11} & \cdot & \sim j & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ f_j x_j & f_{j1} x_j & \cdot & \sim j & \cdot & f_{jn} x_j \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ f_n & f_{n1} & \cdot & \sim j & \cdot & f_{nn} \end{vmatrix}}{\alpha |\boldsymbol{F}|} =$$

(4B. 17) $$\frac{\frac{(-1)^j}{x_j}\begin{vmatrix} f_1 & f_{11} & \cdot & \sim j & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ q & 0 & \cdot & \sim j & \cdot & 0 \\ \cdot & \cdot & \cdot & \sim j & \cdot & \cdot \\ f_n & f_{n1} & \cdot & \sim j & \cdot & f_{nn} \end{vmatrix}}{\alpha |\boldsymbol{F}|} = \frac{(-1)^j q (-1)^{j+1} F_{0,j,j}}{x_j \alpha |\boldsymbol{F}|} =$$

(4B.18) $$\frac{(-1)^{2j+1}(-1)^{2j+1} q_0 x_j x_j |\boldsymbol{F}|}{x_j \alpha q^2 |\boldsymbol{F}|} = \frac{x_j}{\alpha q}$$

or

(4B.19) $$\frac{\partial x_j^*}{\partial q} = \frac{x_j}{q}$$

and

(4B.20) $$\frac{\partial p^*}{\partial w_j} = \frac{x_j}{q}$$

(4B.21) $$\frac{\partial x_i^*}{\partial w_j} = \frac{\begin{vmatrix} 0 & \alpha f_1 & \cdot & 0 & \cdot & \alpha f_n \\ \alpha f_1 & \alpha p f_{11} & \cdot & 0 & \cdot & \alpha p f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \alpha f_j & \alpha p f_{j1} & \cdot & 1 & \cdot & \alpha p f_{jn} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \alpha f_n & \alpha p f_{n1} & \cdot & 0 & \cdot & \alpha p f_{nn} \end{vmatrix}}{\alpha^{n+1} p^{n-1} |\boldsymbol{F}|} = \frac{\alpha^n p^{n-2}\begin{vmatrix} 0 & f_1 & \cdot & 0 & \cdot & f_n \\ f_1 & f_{11} & \cdot & 0 & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_j & f_{j1} & \cdot & 1 & \cdot & f_{jn} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & f_{n1} & \cdot & 0 & \cdot & f_{nn} \end{vmatrix}}{\alpha^{n+1} p^{n-1} |\boldsymbol{F}|} =$$

(4B.22) $$(-1)^{i+j} \frac{|\boldsymbol{F}_{ij}|}{\alpha p |\boldsymbol{F}|} = (-1)^{i+j}(-1)^{i+j} \frac{K_j \sigma_{ij} x_i}{\alpha p f_j}.$$

Using the FOCs,

(4B.23) $$\frac{\partial x_i^*}{\partial w_j} = \frac{K_j \sigma_{ij} x_i}{w_j}$$

(4B.24) $$\frac{\partial p^*}{\partial \alpha} = \frac{\begin{vmatrix} -q & \alpha f_1 & \cdot & \alpha f_n \\ -pf_1 & \alpha p f_{11} & \cdot & \alpha f_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -pf_n & \alpha p f_{n1} & \cdot & \alpha p f_{nn} \end{vmatrix}}{\alpha^{n+1} p^{n-1} |\boldsymbol{F}|} = \frac{-\alpha^n p^n \begin{vmatrix} q & f_1 & \cdot & f_n \\ f_1 & f_{11} & \cdot & f_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ f_n & f_{n1} & \cdot & f_{nn} \end{vmatrix}}{\alpha^{n+1} p^{n-1} |\boldsymbol{F}|} \Rightarrow$$

(4B.25) $$\frac{\partial p^*}{\partial \alpha} = -\frac{p}{\alpha}\frac{|\boldsymbol{F}|}{|\boldsymbol{F}|} = -\frac{p}{\alpha}.$$

Note that $\begin{vmatrix} q & \boldsymbol{f}_{x'} \\ \boldsymbol{f}_x & \boldsymbol{H} \end{vmatrix} = \begin{vmatrix} 0 & \boldsymbol{f}_{x'} \\ \boldsymbol{f}_x & \boldsymbol{H} \end{vmatrix} = |\boldsymbol{F}|$ because adding $-x_i$ times row i (for $i = 1, \ldots, n$) to row 0 in matrix $\boldsymbol{F}$ gives

(4B.26) $$\frac{\partial x_i^*}{\partial \alpha} = \frac{\begin{vmatrix} 0 & \alpha f_1 & \cdot & -q & \cdot & \alpha f_n \\ \alpha f_1 & \alpha p f_{11} & \cdot & -pf_1 & \cdot & \alpha p f_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \alpha f_n & \alpha p f_{n1} & \cdot & -pf_n & \cdot & \alpha p f_{nn} \end{vmatrix}}{\alpha^{n+1} p^{n-1} |\boldsymbol{F}|} = \frac{-\alpha^n p^{n-1} \begin{vmatrix} 0 & f_1 & \cdot & q & \cdot & f_n \\ f_1 & f_{11} & \cdot & f_1 & \cdot & f_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & f_{n1} & \cdot & f_n & \cdot & f_{nn} \end{vmatrix}}{\alpha^{n+1} p^{n-1} |\boldsymbol{F}|}.$$

Using the results from (4A.36) and (4A.37), subtracting column 0 from column i, and factoring gives

(4B.27) $$\frac{\partial x_i^*}{\partial \alpha} = \frac{-q \begin{vmatrix} 0 & f_1 & \cdot & 1 & \cdot & f_n \\ f_1 & f_{11} & \cdot & 0 & \cdot & f_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_n & f_{n1} & \cdot & 0 & \cdot & f_{nn} \end{vmatrix}}{\alpha |\boldsymbol{F}|} = \frac{q}{\alpha}\frac{x_i}{q} = -\frac{x_i}{\alpha} \Rightarrow$$

(4B.28) $$\frac{\partial x_i^*}{\partial \alpha} = -\frac{x_i}{\alpha}.$$

Substituting into (4B.7) yields

(4B.29) $$dp^* = \sum_j \left(\frac{x_j}{q}\right) dw_j - \frac{p}{\alpha} d\alpha \ .$$

Multiply through by $\frac{1}{p}$ and multiplying the first term on the right-hand side by $\frac{w_j}{w_j}$ results in

(4B.30) $$\frac{dp^*}{p} = \sum_j \left(\frac{w_j x_j}{pq}\right) \frac{dw_j}{w_j} - \frac{d\alpha}{\alpha}$$

or

(4B.31) $$E(p) = \sum_j K_j E\left(w_j\right) - E(\alpha).$$

Substituting into (4B.31) into (4B.8) yields

(4B.32) $$dx_i^* = \frac{x_i}{q} dq + \sum_j \left(\frac{K_j \sigma_{ij} \, x_i}{w_j}\right) dw_j - \frac{x_i}{\alpha} d\alpha \ .$$

Multiplying through by $\frac{1}{x_i}$ results in

(4B.33) $$\frac{dx_i^*}{x_i} = \frac{x_i}{x_i q} dq + \sum_j \left(\frac{K_j \sigma_{ij} x_i}{x_i}\right) \frac{dw_j}{w_j} - \frac{d\alpha}{\alpha}$$

or

(4B.34) $$E(x_i) = E(q) + \sum_j K_j \sigma_{ij} E\left(w_j\right) - E(\alpha).$$

»Chapter Five

EDM POLICY APPLICATIONS: ONE OUTPUT, TWO INPUTS

Chapter 4 described how equilibrium displacement models (EDMs) are used to model exogenous shocks to output demand and input supply behavioral equations. An EDM was expanded to accommodate potential differences (i.e., "wedges") between demand and supply prices to test whether the model was homogeneous of degree 0 (HD0) in both input and output prices. In this chapter, we use price and quantity wedges to estimate the impacts of market inventions when prices are not allowed to clear markets at equilibrium levels. This often occurs in response to policy or regulatory shocks. To facilitate using price and quantity wedges, we expand the base EDM by adding equilibrium equations to accommodate policy-induced differences between consumer and producer prices and/or quantities demanded and supplied.

Modeling Exogenous Wedges Caused by Policy Changes

EDMs allow for the evaluation of government policies on output and input quantities and prices. Examples of such policies include excise or sales taxes, wage regulations, and government interventions such as purchases or provision of goods. EDMs can be used to evaluate the impacts of policies that involve multiple interventions, such as simultaneously regulating the price of a good and input usage.

Policy and regulatory actions create wedges between the price that consumers pay and the price that producers receive for a product. In other cases, policies may generate wedges between the price a producer is willing to pay for an input and a supplier's cost of providing the input when, for example, binding price restrictions or quotas on input use are imposed. A third example occurs when a policy causes the desired quantities of a product to differ between consumers and producers (e.g., market interventions that impose price ceilings or floors). For example, binding price ceilings create differences between the quantities that consumers would like to purchase at the resulting lower price and the quantities that suppliers are willing to supply at that price. Conversely, price floors create differences between the quantities that consumers are willing to purchase at the resulting higher price and the quantities that suppliers are willing to supply at that price. In the absence of coercion or other interventions, we observe only one of these quantities in terms of market transactions. EDMs are valuable in this setting because they can be modified to estimate the unobserved levels of quantity or price.

Even if the size of a price wedge (e.g., an excise tax) is known with certainty, the impacts on prices and quantities are endogenous. To accommodate this endogeneity, policy and regulatory actions must be modeled using one or more additional constraints. In many cases, these constraints are represented by equilibrium equations that allow wedges to be placed between demand and supply prices or quantities. In the following discussions, we present modeling procedures and examples for various policy and regulatory actions.

A Tax in the Output Market

Consider the impact of an *ad valorem* sales or excise tax placed on a good or service. Such taxes are commonly imposed on consumer goods—such as gasoline, cigarettes, and alcohol—either to generate revenue for governments or to discourage consumption (Brester et al., 2023). Figure 5.1 illustrates the effect of a per unit excise tax placed on output. Assume that, prior to the tax, the equilibrium price and quantity were p_0 and q_0. Regardless of whether the tax is placed on consumers with a sales tax or suppliers with an excise tax, the tax does not shift either the demand or supply function. That is, the tax neither changes consumer preferences nor production technologies. Such shifts are often graphically presented in economics principles textbooks to illustrate changes in prices and quantities caused by an excise or sales tax. But, this approach is simply a matter of illustrative convenience. Rather, the tax places a wedge, θ_7, between the price paid by consumers, p_1^D, and the price received by producers, p_1^S.

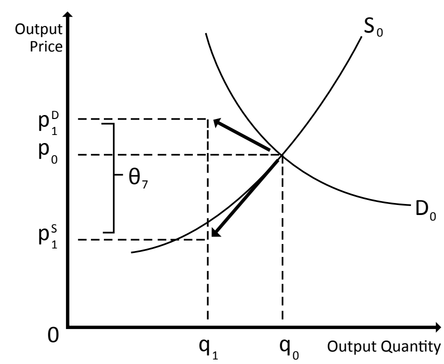

Figure 5.1. An Excise Tax on Output

Although the size of the tax is known with certainty, resulting changes in the producer supply price, the consumer demand price, and the quantity exchanged in the market are endogenous depending upon own-price elasticities of demand and supply. An EDM provides estimates of these changes as movements along linearly approximated equilibrium trajectories, denoted by the arrows in Figure 5.1. The ending points of these trajectories occur where the difference between consumer and producer prices, p^D and p^S, is equal to the size of the imposed tax, θ_7. These ending points are calculated by an EDM after considering all feedback effects across the behavioral equations. The EDM-estimated after-tax equilibrium trajectories lie on tangents to the original total-response supply and demand curves rather than on the curves themselves. The accuracy of the linear approximations depends upon the curvature of the underlying total-response functions and the size of the tax. If the underlying response functions are only moderately nonlinear and/or the tax is relatively small, an EDM can approximate economic outcomes with a reasonable degree of accuracy.

An EDM considers the effects of a sales or excise tax by explicitly incorporating a variable that represents the difference between the affected variables. For example, an excise tax places a wedge between producer price, p^S, and consumer price, p^D, in Figure 5.1, such that $p^D = p^S + \theta_7$, or in proportional elasticity form,

(5.1) $$E(p^D) = E(p^S) + E(\theta_7),$$

where $E(\theta_7) = \frac{d\theta_7}{p_0}$.

This "equilibrium" equation is added to the model presented in (4.29)–(4.34) and explicitly places a price wedge between the consumer demand price, p^D, and the producer supply price, p^S. After making this addition and moving the endogenous variables to the left-hand side, the EDM becomes

(5.2) $$E(q) - \eta^D E(p^D) = E(\theta_1)$$

(5.3) $$E(p^S) - K_1 E(w_1) - K_2 E(w_2) = E(\theta_2)$$

(5.4) $$E(x_1) - E(q) - K_1\sigma_{11}E(w_1) - K_2\sigma_{12}E(w_2) = E(\theta_3)$$

(5.5) $$E(x_2) - E(q) - K_1\sigma_{21}E(w_1) - K_2\sigma_{22}E(w_2) = E(\theta_4)$$

(5.6) $$E(x_1) - \varepsilon_1 E(w_1) = E(\theta_5)$$

(5.7) $$E(x_2) - \varepsilon_2 E(w_2) = E(\theta_6)$$

(5.8) $$E(p^D) - E(p^S) = E(\theta_7).$$

Equations (5.2)–(5.7) are considered behavioral equations, while (5.8) is an equilibrium equation. Using linear algebra, (5.2)–(5.8) is written in matrix notation as

$$
(5.9)\quad \begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 \\ -1 & 0 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} \\ -1 & 0 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} \\ 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p^D) \\ E(p^S) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \end{bmatrix}.
$$

We continue to use Gardner's (1988, p. 99) example to parameterize the model and assume that a 10% excise tax is imposed such that $E(\theta_7) = 0.10$. Consequently, the system becomes

$$
(5.10)\quad \begin{bmatrix} 1 & 0.60 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -0.30 & -0.70 \\ -1 & 0 & 0 & 1 & 0 & 0.70 & -0.70 \\ -1 & 0 & 0 & 0 & 1 & -0.30 & 0.30 \\ 0 & 0 & 0 & 1 & 0 & -0.20 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1.0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q) \\ E(p^D) \\ E(p^S) \\ E(w_1) \\ E(w_2) \\ E(x_1) \\ E(x_2) \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.10 \end{bmatrix}.
$$

Solving (5.10) for the endogenous variables results in

$$
(5.11)\quad \boldsymbol{y} = \begin{bmatrix} E(q) \\ E(p^D) \\ E(p^S) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} -0.032 \\ 0.053 \\ -0.047 \\ -0.013 \\ -0.039 \\ -0.066 \\ -0.039 \end{bmatrix}.
$$

The excise tax causes a 3.2% decrease in the quantity of output $E(q)$; a 5.3% increase in the price consumers pay for the output, $E(p^D)$; and a 4.7% decrease in the price received by producers, $E(p^S)$, relative to the original equilibrium quantity and price. Note that the sum of the absolute values of $E(p^D)$ and $E(p^S)$ represents the 10% wedge imposed by the excise tax.

Although the EDM does not require an estimate of the total-response own-price elasticity of supply for output, ε^S, the inverse of the $\boldsymbol{A}$ matrix that generates the results in (5.11) can be used to calculate it as

$$\varepsilon^S = \frac{E(q)}{E(p^S)} = \frac{-0.032}{-0.047} = 0.68.$$

Because the consumer own-price elasticity of demand ($\eta^D = -0.60$) is more inelastic than the implicit own-price elasticity of supply ($\varepsilon^S = 0.68$), we obtain the expected result that the 10% tax impacts consumer prices, $E(p^D)$, by a larger percentage, 5.3%, than the absolute value of the change in producer prices of 4.7%. The results in (5.11) indicate that the quantity of input 1, $E(x_1)$, declines by 1.3% with a 6.6% concurrent decline in its price, $E(w_1)$. The quantity and price of input 2, $E(x_2)$ and $E(w_2)$, decline by equal percentages (3.9%) because of the assumed unitary input supply elasticity of input 2 (i.e., $\varepsilon_2 = 1.0$).

We emphasize that while the difference between p^D and p^S is determined exogenously by the specified size of the tax, changes in endogenous variable levels cannot be obtained in the absence of equilibrium equation (5.8) for two reasons. First, the tax does not shift either the demand or supply function. Therefore, sales or excise taxes cannot be entered in (5.2) or (5.3)–(5.5) as a shock in demand or production technologies using θ_1 or θ_2–θ_4. Rather, the tax is appropriately included in an EDM by specifying a price wedge between consumer and producer prices, as in (5.8).

Second, while the difference between p_1^D and p_1^S is determined exogenously and with certainty based on the size of the tax, the values of p_1^D and p_1^S are endogenously and simultaneously determined by the EDM. Whenever a policy or regulatory action causes a wedge to be driven between prices paid and received or quantities supplied and demanded, additional equations that represent these price or quantity wedges must be added to an EDM model.

A General Model for Estimating Policy Impacts

Some regulatory policies place wedges between demand and supply prices or quantities in more than one output or input market. A general EDM for quantifying any of these changes can be constructed by adding equations that accommodate potential price or quantity wedges in each market. Therefore, we add the following equilibrium equations to the initial EDM system of (5.2)–(5.7):

(5.12) $E(q^D) = E(q^S) + E(\theta_7)$

(5.13) $E(x_1^D) = E(x_1^S) + E(\theta_8)$

(5.14) $E(x_2^D) = E(x_2^S) + E(\theta_9)$

(5.15) $E(p^D) = E(p^S) + E(\theta_{10})$

(5.16) $E(w_1^D) = E(w_1^S) + E(\theta_{11})$

(5.17) $E(w_2^D) = E(w_2^S) + E(\theta_{12})$.

Note that (5.15) is a restatement of (5.8). Equations (5.12)–(5.17) are referred to as equilibrium equations. The percentage changes in θ_7–θ_{12} represent regulatory or policy-induced changes that introduce wedges between producer and consumer prices or between quantities demanded and supplied. Equation (5.12) allows for a wedge between output demand and supply quantities, while (5.15) allows for a wedge between consumer and producer output prices and costs. Similarly, (5.13) and (5.14) allow for demand and supply quantity wedges in the input markets, while (5.16) and (5.17) allow for price wedges in those markets.

Moving the endogenous variables of (5.12)–(5.17) to the left-hand side and including them with (5.2)–(5.7) yields the following general EDM system of equations:

(5.18) $E(q^D) - \eta^D E(p^D) = E(\theta_1)$

(5.19) $E(p^S) - K_1 E(w_1^D) - K_2 E(w_2^D) = E(\theta_2)$

(5.20) $E(x_1^D) - E(q^S) - K_1\sigma_{11}E(w_1^D) - K_2\sigma_{12}E(w_2^D) = E(\theta_3)$

(5.21) $E(x_2^D) - E(q^S) - K_1\sigma_{21}E(w_1^D) - K_2\sigma_{22}E(w_2^D) = E(\theta_4)$

(5.22) $E(x_1^S) - \varepsilon_1 E(w_1^S) = E(\theta_5)$

(5.23) $E(x_2^S) - \varepsilon_2 E(w_2^S) = E(\theta_6)$

(5.24) $E(q^D) - E(q^S) = E(\theta_7)$

(5.25) $E(x_1^D) - E(x_1^S) = E(\theta_8)$

(5.26) $E(x_2^D) - E(x_2^S) = E(\theta_9)$

(5.27) $E(p^D) - E(p^S) = E(\theta_{10})$

(5.28) $E(w_1^D) - E(w_1^S) = E(\theta_{11})$

(5.29) $E(w_2^D) - E(w_2^S) = E(\theta_{12})$.

Using linear algebra, (5.18)–(5.29) are written in a general form as

(5.30) $\boldsymbol{Ay} = \boldsymbol{b}$,

where $\boldsymbol{A}$ is a 12 × 12 matrix of parameters, $\boldsymbol{y}$ is a 12 × 1 vector of endogenous variables, and $\boldsymbol{b}$ is a 12 × 1 vector of exogenous shocks such that

(5.31)
$$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \end{bmatrix}.$$

After parameterizing the $\boldsymbol{A}$ matrix, the system's endogenous variables are solved for any exogenous shock $\boldsymbol{b}$ as

(5.32) $\boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{b}$.

Homogeneity of Degree 0 in Input and Output Prices

As discussed in Chapter 4, we again conduct the useful exercise of testing for the internal consistency of the EDM system of equations (5.31) by examining the requirement for homogeneity of degree 0 (HD0) in prices. Price homogeneity is tested by simultaneously imposing, for example, a 10% tax on all input prices and a 10% subsidy on output price. The test involves verifying that the EDM predicts no changes in equilibrium output or input quantities. This is accomplished by setting $E(\theta_{11}) = 0.10$, $E(\theta_{12}) = 0.10$, and $E(\theta_{10}) = -0.10$ in (5.31). The input price wedges $E(\theta_{11})$ and $E(\theta_{12})$ are entered as positive numbers because the tax causes input demand prices to be higher than input supply prices. The output price subsidy wedge is entered as a negative number because the subsidy causes the consumer demand price to be 10% lower than the subsidized producer supply price. The resulting $\boldsymbol{b}$ and $\boldsymbol{y}$ vectors become

$$(5.33)\quad \boldsymbol{b} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.10 \\ 0.10 \\ 0.10 \end{bmatrix} \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.10 \\ 0 \\ 0 \\ 0.10 \\ 0.10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The results show that a 10% increase in input demand prices and a simultaneous 10% increase in producer output price causes no change in any other endogenous variable. Consequently, the EDM in (5.31) is HD0 in input and output prices.

Modeling an Output Subsidy

Subsidizing the consumption of a good or service is a common government policy. Examples in the United States include subsidized health insurance premiums for low-income citizens and tax credits for purchases of electric vehicles. When deciding how to model a given policy within an EDM, it is often useful to graphically illustrate the policy to visualize its effects. Figure 5.2 illustrates the general effects of an output subsidy, S_q, while allowing quantities to clear the market. The subsidy drives a wedge between the prices that consumers pay and producers receive for the good or service, with $p_1^D < p_1^S$. Although the size of the subsidy, θ_{10}, is known, the impacts on producer prices, p_1^S, and consumer prices, p_1^D, as well as the quantity of the good or service, q_1, exchanged in the market are endogenous.

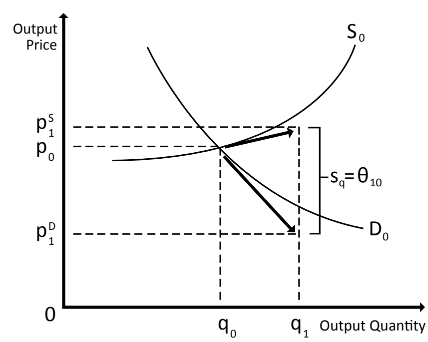

Figure 5.2. A Subsidy on Output

The general EDM presented in (5.18)–(5.29) is used to determine the effect of this subsidy. To operationalize the model, (5.27) is used to place a wedge between the consumer demand price and the producer supply price caused by the subsidy. Assuming a 10% *ad valorem* subsidy, $p^D < p^S$ or $p^D = p^S - \theta_{10}$. The value of $E(\theta_{10})$ in (5.31) is set equal to −0.10 with all other exogenous shock values, $E(\theta_i)$, in vector $\boldsymbol{b}$ set equal to 0.

The EDM is solved using the following vector of exogenous shocks $\boldsymbol{b}$ and results in the vector of changes in endogenous variables $\boldsymbol{y}$:

$$(5.34)\quad \boldsymbol{b} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.10 \\ 0 \\ 0 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} 0.032 \\ -0.053 \\ 0.032 \\ 0.047 \\ 0.013 \\ 0.039 \\ 0.066 \\ 0.039 \\ 0.013 \\ 0.039 \\ 0.066 \\ 0.039 \end{bmatrix}.$$

The results indicate that the new after-subsidy quantity, $E(q^D)$ and $E(q^S)$, exchanged in the market increases by 3.2%. The producer supply price, $E(p^S)$, increases by 4.7%, while the consumer demand price, $E(p^D)$, declines by 5.3%. Note that the sum of the absolute values of these two effects is equal to the size of the 10% subsidy. The producer implicit total-response supply elasticity is identical to the value obtained with an output tax, such that

$$\varepsilon^S = \frac{E(q)}{E(p^S)} = \frac{0.032}{0.047} = 0.68.$$

Consequently, producer prices increase less than consumer prices decline. Hence, producers receive a smaller portion of the subsidy relative to consumers. The price of input 1, $E(w_1^D)$ and $E(w_1^S)$, increases by 6.6% and the use of input 1, $E(x_1^D)$ and $E(x_1^S)$, increases by 1.3%. The price of input 2, $E(w_2^D)$ and $E(w_2^S)$, increases by 3.9% and the use of input 2, $E(x_2^D)$ and $E(x_2^S)$, increases by 3.9%. Changes in the demand and supply input prices and quantities are equivalent because the output price subsidy did not place a price or quantity wedge between these variables.

The percentage responses presented in (5.34) are identical in magnitude to those caused by a 10% excise tax as reported in (5.11). However, the signs on each endogenous variable are opposite for the two policies. The magnitudes are the same for the two scenarios because the results were generated using the same parameterization of the $\boldsymbol{A}$ matrix. But the signs are opposite because the two policies create opposite impacts. In the first case, a 10% *ad valorem* excise tax placed a wedge between consumer demand and producer supply prices, with consumer prices being driven higher and producer prices being driven lower relative to the initial equilibrium point (Figure 5.1). For a 10% *ad valorem* subsidy, the opposite occurs: consumer price decreases and producer price increases relative to the initial equilibrium point (Figure 5.2). The linearity of an EDM causes the absolute value of the impacts of the two policies on the endogenous variables to be identical.

A Tax on Input 1

The full EDM presented in (5.18)–(5.29) can also be used to determine the effect of a tax or subsidy on a factor of production. Excise taxes placed on highway-use diesel fuel in the United States provide an example as diesel fuel is an input for the transportation sector. Consider a case in which a 10% excise tax is placed on input 1. Figure 5.3 shows that the tax places a wedge between the input demand price, w_1^D, and its supply price, w_1^S, with $w_{1,1}^D > w_{1,1}^S$. Thus, $w_{1,1}^D = w_{1,1}^S + \theta_{11}$, indicating that (5.28) is the appropriate mechanism for modeling the tax.

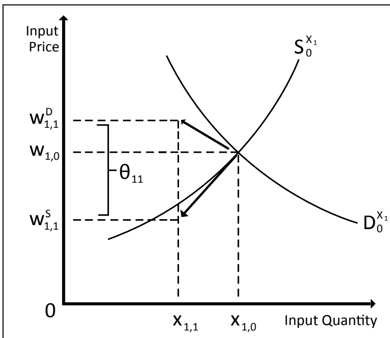

Figure 5.3. A Tax on Input 1

The tax wedge is entered as an exogenous shock in (5.28) by setting $E(\theta_{11}) = 0.10$ in vector ***b*** with all other $E(\theta_i)$ values set equal to 0. The changes in endogenous variables ***y*** are obtained by solving (5.31), as indicated in (5.32), giving:

$$(5.35)\quad \boldsymbol{b} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.10 \\ 0 \end{bmatrix} \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} -0.004 \\ 0.007 \\ -0.004 \\ 0.007 \\ -0.016 \\ 0.001 \\ 0.019 \\ 0.001 \\ -0.016 \\ 0.001 \\ -0.081 \\ 0.001 \end{bmatrix}.$$

The results indicate that the tax placed on input 1 causes the output demand and supply quantities, $E(q^D)$ and $E(q^S)$, to decline by 0.4% while the equilibrium output demand and supply price, $E(p^D)$ and $E(p^S)$, increase by 0.7%. Note that changes in the supply and demand prices and quantities are equivalent because the tax on input 1 does not impose either price or quantity wedges in the output market. The demand price for input 1, $E(w_1^D)$, increases by 1.9% while its supply price, $E(w_1^S)$, declines by 8.1%. The sum of the absolute values of these two changes (8.1% + 1.9%) totals 10%, which was the size of the excise tax placed on input 1. The incidence of the tax borne by sellers and buyers of the input depends upon the own-price elasticities of derived demand and supply of the input. The $\boldsymbol{A}^{-1}$ matrix that generates the results in (5.35) can be used to obtain an estimate of the implied own-price elasticity of the total-response derived demand for input 1 as

$$\eta_{x_1}^{D} = \frac{E(x_1^D)}{E(w_1^D)} = \frac{-0.016}{0.019} = -0.84.$$

Therefore, producers of input 1 bear a larger portion of the 10% tax (8.1%) than the users of the input (1.9%) given that the assumed own-price elasticity of supply of input 1 (0.20) is more inelastic than the implicit derived demand elasticity (−0.84). The tax on input 1 reduces its use, $E(x_1^D)$, by 1.6%. However, because inputs 1 and 2 are substitutes in the production process, both the use of input 2, $E(x_2^D)$, and its price, $E(w_2^D)$, increase (by 0.1%).

A Tax on Input 2

Assume that a 10% tax is placed on input 2. The illustration of this example is analogous to that presented in Figure 5.3, with the appropriate subscripts changed to reflect the second input. The result of this exogenous shock is different than for a 10% tax on input 1 because of differences in the own-price elasticities of supply of the two inputs. The tax places a wedge between the demand price, w_2^D, and supply price, w_2^S, of input 2 so that (5.29) is the appropriate mechanism for implementing the exogenous shock. Once again, since Figure 5.3 indicates that an excise tax causes the demand price for the input to exceed its supply price, the tax is implemented by setting $E(\theta_{12}) = 0.10$ in vector $\boldsymbol{b}$ of (5.31), with all other values entered as 0. The following changes in the endogenous variables are obtained by solving (5.31) as indicated in (5.32):

$$(5.36) \quad \boldsymbol{b} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.10 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} -0.028 \\ 0.046 \\ -0.028 \\ 0.046 \\ 0.003 \\ -0.041 \\ 0.015 \\ 0.059 \\ 0.003 \\ -0.041 \\ 0.015 \\ -0.041 \end{bmatrix}.$$

The results indicate that the output demand and supply quantities, $E(q^D)$ and $E(q^S)$, decline by 2.8% while the equilibrium output demand and supply prices, $E(p^D)$ and $E(p^S)$, increase by 4.6%. The 10% tax placed on input 2 has a larger effect on equilibrium output quantities and price than the effects of a 10% tax placed on input 1 because the own-price elasticity of supply of input 2 is less inelastic than the own-price elasticity of supply of input 1. The tax on input 2 reduces the quantity of the input used, $E(x_2^D)$ and $E(x_2^S)$, by 4.1% and decreases the input supply price $E(w_2^S)$ by 4.1%. The input's derived demand price, $E(w_2^D)$, increases by 5.9%. The higher proportion of the tax paid by users of input 2 is consistent with its implicit derived demand elasticity,

$$\eta_{x_2}^D = \frac{E(x_2^D)}{E(w_S^D)} = \frac{-0.041}{0.059} = -0.70,$$

being more inelastic that the assumed input supply elasticity of $\varepsilon_2 = 1.0$. Because the inputs are substitutes in the production process, the tax on input 2 increases the use of input 1, $E(x_1^D)$, by 0.3% and its demand price, $E(w_1^D)$, by 1.5%. We note that the changes in the demand and supply prices of input 1, $E(w_1^D)$ and $E(w_1^S)$, and the demand and supply quantities, $E(x_1^D)$ and $E(x_1^S)$, are equal because the tax on input 2 did not place a price or quantity wedge in the market for input 1.

Subsidizing Inputs

In lower income countries, governments often subsidize inputs (e.g., fertilizer and seed) used to produce agricultural commodities. In some higher income countries, governments subsidize irrigation water. Because crop insurance is a risk management input used in the production of agricultural commodities, crop insurance premium subsidies represent another input that is subsidized in various countries.

Figure 5.4 shows that a subsidy on input 1, S_x, places a wedge between its demand price, w_1^D, and its supply price, w_1^S, with $w_{1,1}^D < w_{1,1}^S$ such that $w_{1,1}^D = w_{1,1}^S - \theta_{11}$. Equation (5.28) is the appropriate mechanism in the EDM for entering the subsidy, S_x. Assuming a 10% subsidy, a value of –0.10 is entered in (5.31) for $E(\theta_{11})$. The ***b*** vector and the associated solution, ***y***, are

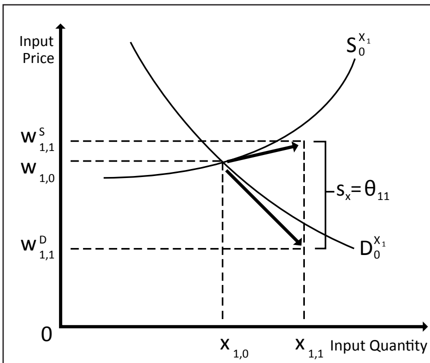

Figure 5.4. Input Subsidy on Input1

$$(5.37)\quad \boldsymbol{b} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.10 \\ 0 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} 0.004 \\ -0.007 \\ 0.004 \\ -0.007 \\ 0.016 \\ -0.001 \\ -0.019 \\ -0.001 \\ 0.016 \\ -0.001 \\ 0.081 \\ -0.001 \end{bmatrix}.$$

The results indicate that the subsidy on input 1 increases demand and supply output quantities, $E(q^D)$ and $E(q^S)$, by 0.4% while output demand and supply prices, $E(p^D)$ and $E(p^S)$, decrease by 0.7%. The demand price, $E(w_1^D)$, for input 1 decreases by 1.9% while the supply price, $E(w_1^S)$, increases by 8.1%. The sum of the absolute values of these two changes (8.1% + 1.9%) equals the size of the 10% subsidy. The subsidy on input 1 increases the equilibrium quantity demanded and supplied of the input, $E(x_1^D)$ and $E(x_1^S)$, by 1.6%. The implied derived demand elasticity is

$$\eta_{x_1}^D = \frac{E(x_1^D)}{E(w_1^D)} = \frac{0.016}{-0.019} = -0.84.$$

Because the inputs are substitutes in the production process, both the use of input 2, $E(x_2^D)$, and its price, $E(w_2^D)$, decrease.

The effect of a 10% subsidy on input 1 on the endogenous variables in (5.37) is the opposite of the effect of a 10% tax on input 1 in (5.35). In addition, the implicit derived demand elasticity, $\eta_{x_1}^D$, remains unchanged (–0.84). This result is caused by the linearity of EDMs and that the only difference in the two exogenous shocks is a positive number entered for $E(\theta_{11})$ in the case of a tax and a negative number in the case of a subsidy. Because the absolute values of these two shocks are equal, the effects on the endogenous variables are of the same magnitude but have opposite signs.

Government Purchases of Output

Consider a case in which a government decides to buy 10% of the output of an industry and then exports it to another country. This action essentially removes 10% of the output from the industry that is being modeled and has been used as a mechanism to increase domestic producer prices. Such policies were common in the European Union and other countries before the general adoption of world trade agreements. Figure 5.5 illustrates the effect of such a government purchase and disposal of amount q^G.

Figure 5.5 indicates that the quantity produced by suppliers, q_1^S, exceeds the amount purchased by consumers, q_1^D, with $q_1^S > q_1^D$ or

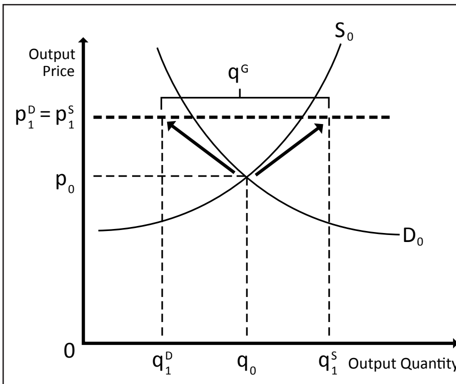

Figure 5.5. Government Purchase and Removal of Output

(5.38) $$q_1^D = q_1^S - q^G = q_1^S - \theta_7.$$

Equation (5.24) is the appropriate equilibrium equation for modeling this policy because it represents the difference between the quantity demanded and the quantity supplied of the product. The EDM is used to calculate the impacts of this policy, assuming a 10% quantity purchase by setting $E(\theta_7) = -0.10$ in vector $\boldsymbol{b}$ of (5.31). The value is entered as a negative number because the policy causes the quantity supplied of the product, q_1^S, to be greater than the domestic quantity demanded, q_1^D, as shown in Figure 5.5. All other values in the vector are set equal to 0. The changes in endogenous variables $\boldsymbol{y}$ are obtained by solving (5.31) as indicated in (5.32), which yields

(5.39) $$\boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} -0.047 \\ 0.079 \\ 0.053 \\ 0.079 \\ 0.022 \\ 0.066 \\ 0.110 \\ 0.066 \\ 0.022 \\ 0.066 \\ 0.110 \\ 0.066 \end{bmatrix}.$$

The changes in the output market indicated in (5.39) are visually indicated by the arrows in Figure 5.5, which represent the equilibrium trajectories of prices and quantities as they move from the initial equilibrium to the new equilibrium as calculated by the EDM. The results indicate that the output demand quantity, $E(q^D)$, declines by 4.7% and the output supply quantity, $E(q^S)$, increases by 5.3%. The sum of the absolute values of the two effects represents the 10% quantity wedge that is being driven between the two values, as indicated by q^G in Figure 5.5. The 10% wedge that is created between quantity demanded and quantity supplied causes the demand output price, $E(p^D)$, and supply output price, $E(p^S)$, to increase by 7.9%. The government purchase causes the demand and supply quantities of input 1, $E(x_1^D)$ and $E(x_1^S)$, to increase by 2.2% and the demand and supply quantities of input 2, $E(x_2^D)$ and $E(x_2^S)$,to increase by 6.6%. The increase in input demands causes the demand and supply prices of input 1, $E(w_1^D)$ and $E(w_1^S)$, to increase by 1.1% and the demand and supply prices of input 2, $E(w_2^D)$ and $E(w_2^S)$, to increase by 6.6%.

Modeling Endogenous Wedges Caused by Policy Changes

The preceding examples were operationalized by shocking a single equilibrium equation. This approach is used whenever a policy places a known wedge between input or output prices or between quantities demanded and supplied. However, some policies must be modeled using endogenous price or quantity wedges or a combination of exogenous and endogenous wedges. Endogenous price or quantity wedges require that additional restrictions and new variables, shadow values, be included in an EDM for the purpose of system identification. The specific restriction that is added depends upon the issue being addressed. That is, a policy-induced restriction on prices or quantities may necessitate constructing one or more restriction equations, which allows the model to endogenously compute the level of the imposed price or quantity wedge, $E(\theta_i)$. This is accomplished by reclassifying the associated wedge, $E(\theta_i)$, as endogenous, moving $E(\theta_i)$ to the left-hand side of the system of equations (5.22)–(5.29), and adding the required restriction equation to the system.

For example, the United States used target prices on grain production during the 1970s and 1980s as a means of increasing domestic producer prices. Suppose a target price is set 10% above an initial equilibrium price and that markets are allowed to clear resulting quantity levels. Figure 5.6 illustrates a case in which a government commits to pay producers a per unit payment equal to the difference between the target price and market price. Because the target price guarantee is above the market equilibrium price, industry output increases from q_0 to q_1. However, the policy places a

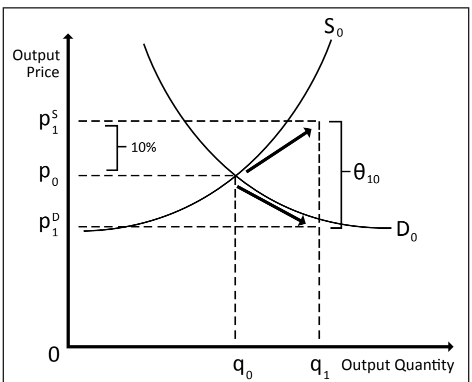

Figure 5.6. A Target Price on Output

wedge between the prices that producers receive for their commodity—the target price, p_1^S—and the price that consumers pay, p_1^D, for the commodity at the higher level of output, q_1. The difference, or wedge, between these prices is endogenous depending upon relative supply and demand elasticities. That is, both the quantity produced and the consumer price needed to establish a quantity equilibrium that removes the additional production from the market are endogenous to the system.

To accommodate this endogeneity, the policy must be modeled with an additional restriction. The additional restriction requires that a new endogenous parameter be added so the system can be mathematically identified. The restriction and added endogenous parameter are specific to the issue being addressed. In the case of a target price established above the equilibrium price, a price wedge is created between p^D and p^S, with $p_1^D < p_1^S$ or $p_1^D = p_1^S - \theta_{10}$ in (5.27). However, the size of this wedge is an endogenous variable so that (5.27) is now written as

$$(5.40) \quad E(p^D) - E(p^S) + E(\theta_{10}) = 0.$$

To implement the target price policy, an additional equation must be added. That is, the target price producers receive is $p^S = p_0 + \psi_1$, so that

$$(5.41) \quad E(p^S) = E(\psi_1),$$

where $E(\psi_1) = \frac{\psi_1}{p_0}$.

The new system of equations now consists of (5.18)–(5.26), (5.40), (5.28), (5.29), and (5.41) such that (5.31) becomes

$$(5.42) \quad \begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ 0 \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix}.$$

Changes in the endogenous variables are obtained by entering the appropriate values in vector $\boldsymbol{b}$ of (5.42). The target price policy is implemented by setting $E(\psi_1) = 0.10$ in vector $\boldsymbol{b}$ and all other values in the vector to 0. The following changes in the endogenous variables are obtained by solving (5.42), as noted in (5.32):

$$(5.43) \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} 0.067 \\ -0.111 \\ 0.067 \\ 0.100 \\ 0.028 \\ 0.083 \\ 0.139 \\ 0.083 \\ 0.028 \\ 0.183 \\ 0.139 \\ 0.083 \\ 0.211 \end{bmatrix}.$$

Note that the quantities produced, $E(q^S)$, and consumed, $E(q^D)$, increase by 6.7%. The price paid by consumers, $E(p^D)$, declines by 11.1% and the price received by producers, $E(p^S)$, increases by the mandated 10%. The target price policy creates a wedge between producer and consumer prices equal to 21.1% of the original equilibrium price, which is the sum of the increase in producer price and the absolute value of the reduction in consumer price. This value is determined endogenously by the model and is represented by the shadow value of $E(\theta_{10})$. The demand and supply quantities, $E(x_1^D)$ and $E(x_1^S)$, for input 1 increase by 2.8% and its demand, $E(w_1^D)$, and supply, $E(w_1^S)$, prices increase by 13.9%. The demand and supply quantities for input 2, $E(x_2^D)$ and $E(x_2^S)$ increase by 8.3%; its demand and supply prices, $E(w_2^D)$ and (w_2^S), also increase by 8.3%. Note that to maintain the target price while allowing quantities to clear the market, the government pays producers 10% more than the original equilibrium price, which is 21.1% more than the new price that consumers pay.

A Price Floor in the Output Market: No Government Purchases

The imposition of price floors is a common government policy used to support the prices of various goods and services, particularly for agricultural commodities. Figure 5.7 presents a situation in which a price floor is imposed at price p_1^D. At this price, a wedge, q^F, exists between consumers' and producers' desired quantities of the good. At price p_1^D, consumers are only willing to purchase quantity q_1^D, while producers would like to produce q_1^S. The policy also creates a price wedge between the price consumers are willing to pay, p_1^D, for quantity q_1^D and the marginal cost to producers, p_1^S, of supplying the lower quantity. The lower level of output, q_1^D, is the quantity actually exchanged in the market, assuming that consumers cannot be forced to buy goods and services beyond their willingness to pay for them.

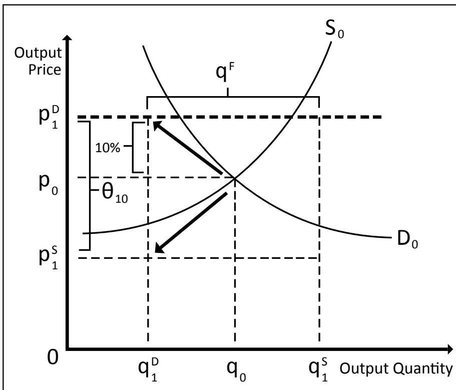

Figure 5.7. A Price Floor on Output with No Government Purchases

Assuming quantities are allowed to clear the market without government purchases, the price floor creates an unknown wedge between $E(p_1^D)$ and $E(p_1^S)$ in (5.27). Therefore, (5.27) is used to implement the policy by allowing $E(\theta_{10})$ to be an endogenous variable and is now written as

$$(5.44) \quad E(p^D) - E(p^S) - E(\theta_{10}) = 0.$$

An additional equation is added to the EDM to represent the percentage change in the equilibrium price at the legislated price floor. If the price floor is established 10% above the equilibrium price, consumers must purchase the product at the legislated higher price. However, the quantity purchased is determined by consumer demand. The policy is included in the model by adding the restriction

$$(5.45) \quad E(p^D) = E(\psi_1).$$

The price floor reduces the quantity supplied from q_0 to q_1^D, as illustrated in Figure 5.7. Although producers would want to increase production at the price floor, p_1^D, from q_0 to q_1^S, consumers cannot be forced to purchase that quantity at the higher price. Consequently, unless a government is willing to purchase the potential surplus production and either store or destroy it, rational producers recognize that they cannot sell all they would like to produce at the higher price. Therefore, q_1^D would be the actual quantity supplied rather than q_1^S.

The new system of equations now consists of (5.18)–(5.26), (5.44), (5.28), (5.29), and (5.45) such that (5.31) becomes

$$(5.46)\quad \begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ 0 \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix}.$$

The price floor policy is implemented by setting $E(\psi_1) = 0.10$ in vector $\boldsymbol{b}$ because the price consumers pay for the reduced output is 10% greater than the original equilibrium price. All other $E(\theta_i)$ values are set equal to 0. The following changes in the endogenous variables are obtained by solving (5.46), as noted in (5.32):

$$(5.47)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} -0.060 \\ 0.100 \\ -0.060 \\ -0.090 \\ -0.025 \\ -0.075 \\ -0.125 \\ -0.075 \\ -0.025 \\ -0.075 \\ -0.125 \\ -0.075 \\ 0.190 \end{bmatrix}.$$

Note that the quantity of output consumed, $E(q^D)$, and produced, $E(q^S)$, declines by 6.0% while the price floor increases consumer prices, $E(p^D)$, by 10%. The marginal cost of providing the lower quantity $E(p^S)$ decreases by 9%. The price floor creates a price wedge between the amount that consumers pay for the product and the marginal cost of producing this lower quantity. The difference equals 19% of the original equilibrium price. This is the shadow value indicated by $E(\theta_{10})$

and is also the sum of the absolute value of the change in the output demand price and the output supply price. The quantity of input 1 used, $E(x_1^D)$ and $E(x_1^S)$, decreases by 2.5% and its price, $E(w_1^D)$ and $E(w_1^S)$, declines by 12.5%. The quantity of input 2 used, $E(x_2^D)$ and $E(x_2^S)$, decreases by 7.5% and its price, $E(w_2^D)$ and (w_2^S), decreases by 7.5%.

A Price Floor in the Output Market: Government Purchases of Surplus Production

The imposition of a price floor is modeled differently if a surplus is allowed to be produced and a government removes it from the market. This was essentially an approach for supporting grain prices in the United States through various nonrecourse loan policies until 1996. In many lower income economies, governments often provide a minimum guaranteed price for agricultural commodities that is above a market equilibrium price. Such policies represent a price floor. Further, it is not uncommon for a government to purchase or auction surplus commodities. Governments sometimes directly distribute commodities to processors and consumers. For example, India's Agricultural Produce Market Committee establishes a minimum price for some agricultural commodities and auctions the surplus at or above that price with a variety of restrictions on who can make purchases.

Price floors cause producers to increase production, but domestic and foreign consumers are not willing to purchase the increased production at the higher floor price. Therefore, governments often purchased surplus domestic agricultural commodities and exported them at lower prices to other countries. In some cases, governments paid for commercial storage to remove production, at least temporarily, from the market so that the floor price remained effective.

Figure 5.8 presents a situation in which a price floor is imposed at price p_1^D. The policy creates a quantity wedge between the quantity demanded by consumers, q_1^D, and the quantity that producers are willing to supply, q_1^S, at the higher floor price. The quantity wedge represents a surplus of size q^F.

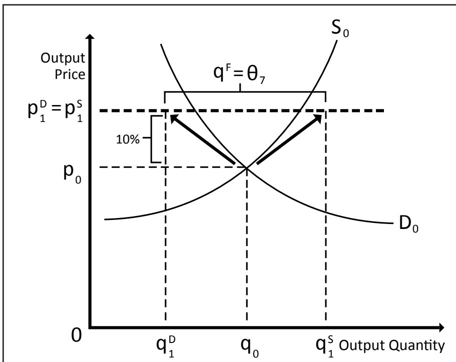

Figure 5.8. A Price Floor on Output with Government Purchases

For this example, the price floor is modeled as an unknown quantity wedge between q_1^D and q_1^S, with $q_1^D < q_1^S$ or $q_1^D = q_1^S - \theta_7$. Therefore, (5.24) is the appropriate equation to alter with $E(\theta_7)$ becoming an endogenous variable so that it is now written as

(5.48) $E(q^D) - E(q^S) + E(\theta_7) = 0.$

An additional equation is added to the EDM that represents the percentage increase in the legislated price floor above the equilibrium price. This effect is included in the model by adding the restriction

(5.49) $E(p^S) = E(\psi_1).$

The price floor signals producers to increase output from q_0 to q_1^S, as shown in Figure 5.8. That result, however, only occurs if the domestic government agrees to buy the surplus. Otherwise, the price floor is difficult to maintain in the face of higher production because consumers will only purchase q_1^D at this higher price. The new system of equations is (5.18)–(5.23), (5.48), (5.25)–(5.29), and (5.49) such that (5.31) becomes

$$(5.50)\quad \begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_7) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ 0 \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix}.$$

Changes in the endogenous variables are obtained by entering the appropriate values in vector ***b*** of (5.50). For a price floor established at 10% above the initial equilibrium price, the policy is implemented by setting $E(\psi_1) = 0.10$ as producer prices will be 10% above the original equilibrium price. All other values in the vector are set equal to 0. The following changes in the endogenous variables ***y*** are obtained by solving (5.50) as shown in (5.32):

$$\text{(5.51)} \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_7) \end{bmatrix} = \begin{bmatrix} -0.060 \\ 0.100 \\ 0.067 \\ 0.100 \\ 0.028 \\ 0.083 \\ 0.139 \\ 0.083 \\ 0.028 \\ 0.083 \\ 0.139 \\ 0.083 \\ 0.127 \end{bmatrix}.$$

Note that quantity, $E(q^D)$, declines by 6% because the price floor has increased consumer prices, $E(p^D)$, by 10%. This is the same result as obtained in (5.50), in which the same price floor was established. However, the remaining results differ as more production occurs given the government purchase of the surplus. The quantity supplied, $E(q^S)$, increases by 6.7% because the price received by producers, $E(p^S)$, has increased by 10%. The price floor creates a quantity wedge of 12.7% between the amount that consumers are willing to purchase at the higher price and the amount that producers supply. This is the shadow value indicated by $E(\theta_7)$ and is also equal to the sum of the absolute value of the change in the output demand, $E(q^D)$, and supply, $E(q^S)$, quantities. The quantity of input 1 used, $E(x_1^D)$ and $E(x_1^S)$, increases by 2.8% and its demand and supply price increases by 13.9%. The quantity of input 2 used, $E(x_2^D)$ and $E(x_2^S)$, and its price increases by 8.3%.

A Price Ceiling in the Output Market with Quantity Clearing

The imposition of price ceilings is a common government policy used to lower the market prices of goods and services. Rent controls in various U.S. cities are examples of price ceilings designed to lower the price of rental housing. Price ceilings are often imposed on food or agricultural commodities in many lower income countries.

Figure 5.9 depicts the impact of a price ceiling that is set below the equilibrium price, p_0. Consequently, the ceiling price represents the price, p_1^S, that producers receive for the good or service. At the ceiling price, the quantity demanded, q_1^D, exceeds the quantity supplied, q_1^S, which creates a shortage. The shortage of goods or services necessitates some type of rationing scheme because, at lower prices, the quantity demanded for such goods and services exceeds the quantity supplied. If prices are not allowed to allocate scarce goods or resources, a nonprice allocation mechanism such as ration coupons, queuing, nepotism, or favoritism must be used as a rationing device.

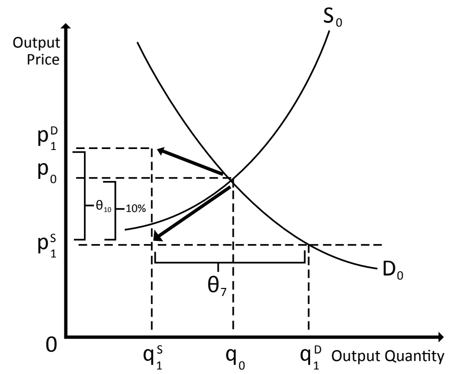

Figure 5.9. A Price Ceiling in the Output Market

Price ceilings cannot be modeled as a wedge between the quantities demanded and supplied of the good at the lower ceiling price because, unless producers are forced to supply more of the product at the lower price, they are only willing to supply q_1^S. Therefore, the price ceiling is modeled as a wedge between the price consumers are willing to pay, p_1^D, for the lower quantity, q_1^S, and the amount they actually pay, p_1^S, in Figure 5.9. Therefore, (5.27) is rewritten as

$$(5.52)\quad E(p^D) - E(p^S) - E(\theta_{10}) = 0$$

so that the size of the price wedge $E(\theta_{10})$ becomes an endogenous variable. An additional equation is added to the EDM to represent the percentage that the price ceiling is below the equilibrium price:

$$(5.53)\quad E(p^S) = E(\psi_1).$$

The new EDM system of equations now consists of (5.18)–(5.26), (5.52), (5.28), (5.29), and (5.53) such that equation (5.31) becomes

$$(5.54)\quad \begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ 0 \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix}.$$

Changes in the endogenous variables are obtained by entering the appropriate values in vector $\boldsymbol{b}$ of (5.54). The price ceiling is implemented by setting $E(\psi_1) = -0.10$ in vector $\boldsymbol{b}$ because the ceiling reduces producer prices by 10% below the original equilibrium price. All other values in the vector are set to 0. The following changes in the endogenous variables are obtained by solving (5.54) as noted in (5.32), giving

$$\text{(5.55)} \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} -0.067 \\ 0.111 \\ -0.067 \\ -0.100 \\ -0.028 \\ -0.083 \\ -0.139 \\ -0.083 \\ -0.028 \\ -0.083 \\ -0.139 \\ -0.083 \\ 0.211 \end{bmatrix}.$$

Note that the quantity produced, $E(q^S)$, is reduced by 6.7% because the price ceiling reduced the price that producers receive, $E(p^S)$, by 10%. Consequently, quantity consumed, $E(q^D)$, also declines by 6.7%. The price that consumers would be willing to pay for the lower output, $E(p^D)$, is 11.1% more than the original equilibrium price. Hence, the price ceiling places a wedge of 21.1% between the price that producers receive and the price that consumers who have access to the limited output are willing to pay. This wedge is indicated by the shadow value $E(\theta_{10})$. The price ceiling reduces the quantity of input 1 used, $E(x_1^D)$ and $E(x_1^S)$, by 2.8% and its demand and supply price, $E(w_1^D)$ and $E(w_1^S)$, decreases by 13.9%. The use of input 2, $E(x_2^D)$ and $E(x_2^S)$, and its price, $E(w_2^D)$ and $E(w_2^S)$, decreases by 8.3%.

Price Ceiling in the Output Market: Size of Shortages

A price ceiling creates a shortage in the output market. Some governments respond to this shortage through actions such as building public housing in rent-controlled areas or by purchasing goods from other countries and placing them into the domestic market. It may be of interest to policy makers to have an estimate of the size of these actions needed to offset the detrimental supply-side effects of a price ceiling. The size of the shortage is endogenous and represented by θ_7 in Figure 5.9.

For this problem, the shortage is modeled as the difference between the quantity demanded and supplied of the good or service. An endogenous quantity wedge is placed between these two variables as illustrated in Figure 5.9, and (5.24) is the appropriate equation to model this market intervention as

$$E(q^D) - E(q^S) - E(\theta_7) = 0, \tag{5.56}$$

such that $E(\theta_7)$ becomes an endogenous variable. A restriction is added to the EDM to represent the price ceiling. Because the price ceiling is imposed on producers, the restriction is written as

$$E(p^S) = E(\psi_1). \tag{5.57}$$

The new system now consists of equations (5.18)–(5.23), (5.56), (5.25)–(5.29), and (5.57) such that (5.31) becomes

$$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_7) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ 0 \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix}. \tag{5.58}$$

Changes in the endogenous variables are obtained by entering the appropriate values in vector ***b*** of (5.58). The price ceiling is implemented by setting $E(\psi_1)$ = –0.10 because the ceiling causes the producer price to be 10% less than the original equilibrium price. All other values in the vector are set equal to 0. The following changes in the endogenous variables ***y*** are obtained by solving (5.58) as noted in (5.32):

$$(5.59)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_7) \end{bmatrix} = \begin{bmatrix} 0.060 \\ -0.100 \\ -0.067 \\ -0.100 \\ -0.028 \\ -0.083 \\ -0.139 \\ -0.083 \\ -0.028 \\ -0.083 \\ -0.139 \\ -0.083 \\ 0.127 \end{bmatrix}.$$

Note that the quantity produced, $E(q^S)$, is reduced by 6.7% because of the price ceiling, which is set 10% below the initial equilibrium price. This is the same result as that generated in the previous example. The quantity that consumers would be willing to purchase, $E(q^D)$, at the 10% lower price increases by 6.0%. Hence, a price ceiling with an anticipated government production of goods or services places a wedge of 12.7% between the amount that producers supply at the ceiling price and the amount that consumers want to purchase at that price. The size of this shortage is indicated by the shadow value, $E(\theta_7)$, which equals 12.7%. The remaining endogenous variables are affected by amounts identical to those obtained in the preceding example.

Price Floor on Input 1

It is common for governments to impose price floors in input markets to support various input prices. Minimum hourly wage laws provide a classic example in which governments do not allow labor to be hired below a specific wage level (e.g., Hamilton et al., 2020). Assume that a price floor on input 1 is legislated such that it is 10% above the equilibrium price, as illustrated in Figure 5.10. The observed market price, $w_{1,1}^D$, reflects the input users' willingness to pay for, and the marginal value of, input 1. Because the input price is 10% higher than the initial equilibrium price, $w_{1,0}$, the quantity demanded of the

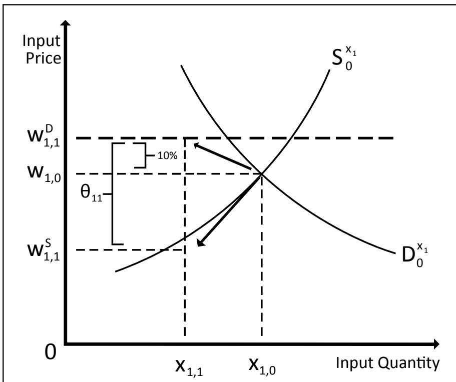

Figure 5.10. A Price Floor on Input 1

input is lower than the initial equilibrium quantity, $x_{1,0}$. Therefore, the marginal cost, or willingness-to-receive price, $w^S_{1,1}$, is less than the initial equilibrium price.

A minimum wage policy places an unknown wedge between the demand price, $E(w^D_1)$, for input 1 and the marginal cost of providing input 1, $E(w^S_1)$, because of the reduction in quantity demanded. This wedge is included in the EDM by altering (5.28) such that $E(\theta_{11})$ becomes an endogenous variable:

$$E(w^D_1) - E(w^S_1) - E(\theta_{11}) = 0. \tag{5.60}$$

An additional restriction is added to the EDM to reflect the policy shock on the demand price of the input and is the mechanism for imposing the price floor:

$$E(w^D_1) = E(\psi_1). \tag{5.61}$$

The new EDM system consists of equations (5.18)–(5.27), (5.60), (5.29), and (5.61) such that (5.31) becomes

$$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x^D_1) \\ E(x^D_2) \\ E(w^D_1) \\ E(w^D_2) \\ E(x^S_1) \\ E(x^S_2) \\ E(w^S_1) \\ E(w^S_2) \\ E(\theta_{11}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ 0 \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix}. \tag{5.62}$$

After parameterizing the $\boldsymbol{A}$ matrix using the values noted above, $E(\psi_1)$ is set equal to 0.10 in vector $\boldsymbol{b}$ because the legislated value of w^D_1 is 10% larger than its equilibrium value. All other entries are set equal to 0 in the vector. The solution to (5.62) yields the following estimates of changes in the endogenous variables:

$$\text{(5.63)} \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{11}) \end{bmatrix} = \begin{bmatrix} -0.021 \\ 0.035 \\ -0.021 \\ 0.035 \\ -0.086 \\ 0.007 \\ 0.100 \\ 0.007 \\ -0.086 \\ 0.007 \\ -0.430 \\ 0.007 \\ 0.530 \end{bmatrix}.$$

The price floor placed on input 1 reduces its use, $E(x_1^D)$, by 8.6%. The price of input 1, $E(w_1^D)$, increases by 10% because the imposed price floor was 10% above the original equilibrium price. However, the supply price, $E(w_1^S)$, or marginal cost of input 1 is reduced by 43% because the opportunity cost of the last unit of input 1 used in the production process is 43% lower than its initial equilibrium value. Therefore, the price ceiling places a price wedge, $E(\theta_{11})$, of 53% between the demand and supply prices of input 1 relative to the initial equilibrium price. This relatively large value results from the assumption that the own-price elasticity of supply of input 1 is relatively inelastic ($\varepsilon_1 = 0.20$).

The quantity of output produced and consumed, $E(q^S)$ and $E(q^D)$, is reduced by 2.1% and the output demand and supply price, $E(p^D)$ and $E(p^S)$, increases by 3.5%. Although output declines, the reduction in the use of input 1 causes more of its substitute, input 2, to be used, as $E(x_2^D)$ and $E(x_2^S)$ increase by 0.7%. In addition, the price of input 2, $E(w_2^D)$ and $E(w_2^S)$, increases by 0.7%.

Price Ceiling on Input 2

Government policies may limit the price at which a productive input can be sold. For example, the prices of electricity and transportation services (e.g., the operating costs of buses or railroads) are limited by legislative actions in many countries. Figure 5.11 presents an example in which a price ceiling has been placed on the suppliers of input 2 so that the supply price, $w_{2,1}^S$, is 10% below its equilibrium price of $w_{2,0}$.

This policy places an unknown wedge between the input 2 demand price, w_2^D, and its supply price, w_2^S, which represents the marginal cost (and value) of the input to the economic system. Further, in the absence of coercion, input suppliers ultimately determine the amount of the input supplied. Therefore, (5.29) is altered so that $E(\theta_{12})$ becomes an endogenous variable and is now written as

(5.64) $$E(w_2^D) - E(w_2^S) - E(\theta_{12}) = 0.$$

An additional restriction is added to the system of equations to represent the reduction in the supply price imposed on producers of the input:

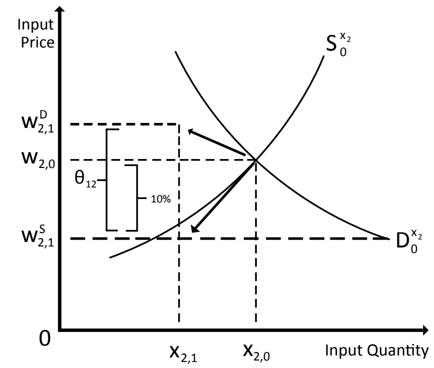

Figure 5.11. A Price Ceiling on Input 2

(5.65) $$E(w_2^S) = E(\psi_1),$$

which represents the price ceiling. The new system of equations is (5.18)–(5.28), (5.64), and (5.65) such that (5.31) becomes

(5.66)
$$\begin{bmatrix} 1 & -\eta_D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ 0 \\ E(\psi_1) \end{bmatrix}.$$

After parameterizing the ***A*** matrix and setting $E(\psi_1)$ = −0.10 and all other $E(\theta_i)$ equal to 0 in ***b*** vector, the solution to (5.66) yields the following estimates of changes in the endogenous variables:

$$\text{(5.67)}\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} -0.068 \\ 0.113 \\ -0.068 \\ 0.113 \\ 0.008 \\ -0.100 \\ 0.038 \\ 0.145 \\ 0.008 \\ -0.100 \\ 0.038 \\ -0.100 \\ 0.245 \end{bmatrix}.$$

The price ceiling on input 2 reduces its supply price, w_2^S, by 10% relative to its initial equilibrium price. The amount of input 2 used and produced, $E(x_2^D)$ and $E(x_2^S)$, declines by 10%. The equivalency of the two results is caused by the use of 1.0 as the own-price elasticity of supply of input 2. The size of the wedge between the demand and supply price of input 2, $E(\theta_{12})$, is 24.5% of the original equilibrium price. The input supply price actually received, $E(w_2^S)$, is 10% below the initial equilibrium price, while the EDM system's marginal value or willingness to pay for the input is its demand price, $E(w_2^D)$, which is 14.5% above the original equilibrium price. Hence, the sum of the absolute value of these two estimates is equal to the indicated shadow value, $E(\theta_{12})$, of 24.5%.

The reduction in the use of input 2 causes the amount of output produced and consumed, $E(q^S)$ and $E(q^D)$, to decline by 6.8% while the price of output, $E(p^D)$ and $E(p^S)$, increases by 11.3%. The legislated price ceiling on input 2 increases the use of its substitute input even though overall production is reduced. The use of input 1, $E(x_1^D)$, increases by 0.8%, while its price, $E(w_1^D)$, increases by 3.8%.

Regulation of Output

Various policies have been legislated to limit the production of a good or service. In many cases, these policies are imposed as a means for reducing negative externalities (e.g., a cap on greenhouse gases or other emissions). Assume that a 10% reduction in the output of an industry has been legislated (the bold vertical dashed line in Figure 5.12), which creates an unknown wedge between p^D and p^S.

Equation (5.27) is altered so that $E(\theta_{10})$ becomes an endogenous variable:

$$\text{(5.68)}\quad E(p^D) - E(p^S) - E(\theta_{10}) = 0.$$

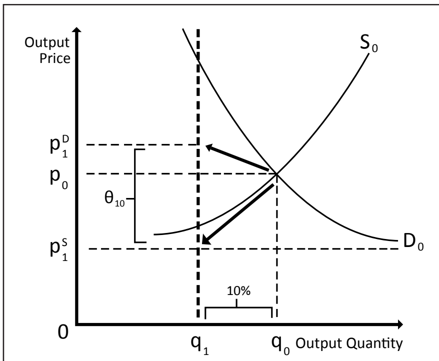

Figure 5.12. A Restriction on Output

An additional restriction is added to the system of equations to represent the reduction in the supply quantity that is imposed on producers of the output:

$$E(q^S) = E(\psi_1). \tag{5.69}$$

The new system of equations is (5.18)–(5.26), (5.68), (5.28), (5.29), and (5.69) such that (5.31) becomes

$$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ 0 \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix}. \tag{5.70}$$

After parameterizing the ***A*** matrix and setting $E(\psi_1) = -0.10$ and all other $E(\theta_i)$ entries in vector ***b*** equal to 0, the solution to (5.70) yields the following estimates of changes in the endogenous variables:

$$(5.71) \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} -0.100 \\ 0.167 \\ -0.100 \\ -0.150 \\ -0.042 \\ -0.125 \\ -0.208 \\ -0.125 \\ -0.042 \\ -0.125 \\ -0.208 \\ -0.125 \\ 0.317 \end{bmatrix}.$$

The 10% legislated reduction in output is represented by 10% reductions in the quantity of output produced, $E(q^S)$, and consumed, $E(q^D)$. The consumer price for the output, $E(p^D)$, increases by 16.7% above the original equilibrium price while the producer price, $E(p^S)$—which represents the marginal cost of production at the reduced output level—declines by 15%. The legislated reduction in output places a price wedge of 31.7% of the original equilibrium price between these two prices, which the model provides as the shadow value, $E(\theta_{10})$. The value of $E(\theta_{10})$ can also be interpreted as the size of a per unit output tax (relative to the initial equilibrium price) that would be needed to achieve the desired 10% reduction in output. The restricted level of output reduces the use of input 1, $E(x_1^D)$, by 4.2% and its price, $E(w_1^D)$, by 20.8%. The use of input 2, $E(x_2^D)$, declines by 12.5% and its price, $E(w_2^D)$, decreases by an equal amount.

Restrictions on the Use of Input 1

Another approach to reducing negative externalities is to legislate a reduction in the use of an input. For example, it is common to limit the use of certain crop protectants, land area, crop nutrients, water, or other inputs in the production of agricultural commodities. Assume that the use of input 1 is legislatively restricted to be 10% lower than its equilibrium level, which is illustrated in Figure 5.13 by the bold vertical dashed line. This policy cre-

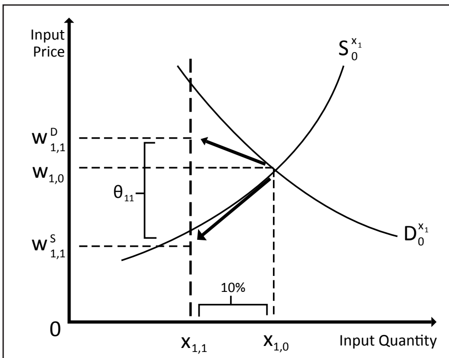

Figure 5.13. A Restriction on the Quantity of Input 1

ates an unknown wedge between $E(w_1^D)$ and $E(w_1^S)$. Therefore, $E(\theta_{11})$ becomes an endogenous variable in (5.28) such that

$$E(w_1^D) - E(w_1^S) - E(\theta_{11}) = 0. \tag{5.72}$$

The quantity restriction that is added to the system of equations is given by

$$E(x_1^D) = E(\psi_1). \tag{5.73}$$

The new system of equations now consists of (5.18)–(5.27), (5.72), (5.29), and (5.73) such that (5.31) becomes:

$$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{11}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ 0 \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix}. \tag{5.74}$$

After parameterizing the $\boldsymbol{A}$ matrix, the 10% quantity restriction on input 1 is introduced by setting $E(\psi_1) = -0.10$ and all other $E(\theta_i)$ entries to 0 in vector $\boldsymbol{b}$. The solution to (5.74) yields the following changes in the endogenous variables:

$$\boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{11}) \end{bmatrix} = \begin{bmatrix} -0.024 \\ 0.041 \\ -0.024 \\ 0.041 \\ -0.100 \\ 0.008 \\ 0.116 \\ 0.008 \\ -0.100 \\ 0.008 \\ -0.500 \\ 0.008 \\ 0.616 \end{bmatrix}. \tag{5.75}$$

The legislated 10% reduction in the use of input 1 is represented by the value of $E(x_1^D)$, and the restriction increases the demand price of input 1 by 11.6%. The marginal cost of supplying input 1 is reduced by 50% as indicated by $E(w_1^S)$. Note that the sum of the absolute values of these two changes is identical to $E(\theta_{11})$ or 61.6%, which represents the endogenously determined wedge between the demand price and the supply price of input 1. The results show that the same 10% reduction in the use of input 1, $E(\theta_{11})$, could have been obtained by imposing a 61.6% tax on this input. However, this value would not be known prior to estimating the EDM. The legislated 10% reduction in the use of input 1 reduces the quantity of output produced and consumed, $E(q^S)$ and $E(q^D)$, by 2.4%. In addition, the price of the output, $E(p^D)$ and $E(p^S)$, increases by 4.1%. The use of input 2, $E(x_2^D)$, increases by 0.8% and its price, $E(w_2^D)$, increases by 0.8%.

Policies Involving Two or More Simultaneous Restrictions

Most policy makers operate in a multidimensional policy and regulatory environment. For example, when the U.S. government implemented target price programs (see Figure 5.6), it simultaneously required producers to reduce planted acreages (i.e., a set-side requirement) if they wanted to receive the price subsidy. The purpose of the restriction was to reduce production and avoid a surplus so that the target price could be maintained with relatively low government outlays. The set-aside provision can be modeled as an input restriction, as in Figure 5.13. An EDM can be constructed to include both provisions by simultaneously adding two wedges using two additional restrictions. The restrictions are imposed using equations (5.40), (5.60), and (5.41) and adding an equation that represents the restriction on input 1:

(5.76) $E(p^D) - E(p^S) + E(\theta_{10}) = 0$

(5.77) $E(w_1^D) - E(w_1^S) - E(\theta_{11}) = 0$

(5.78) $E(p^S) = E(\psi_1)$

(5.79) $E(x_1^D) = E(\psi_2).$

Equation (5.76) places an endogenous price wedge, $E(\theta_{10})$, between consumer and producer prices, which represents the target price policy, while (5.77) places an endogenous price wedge, $E(\theta_{11})$, between the demand and supply prices of the restricted input. For our example, planted acreage is the restricted input that is sufficient to reduce the use of input 1 by the targeted amount. Assume that the producer target price, $E(p^S)$, is set 10% above the initial equilibrium price so that $E(\psi_1) = 0.10$. Furthermore, assume that the program requires a concurrent 10% reduction in the use of input 1, $E(x_1^D)$. This is operationalized by setting $E(\psi_2) = -0.10$. The new EDM system of equations is now (5.18)–(5.26), (5.76), (5.77), (5.29), (5.78), and (5.79) such that (5.31) becomes

$$(5.80)\quad \begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \\ E(\theta_{11}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ 0 \\ 0 \\ E(\theta_{12}) \\ E(\psi_1) \\ E(\psi_2) \end{bmatrix}.$$

After parameterizing the $\boldsymbol{A}$ matrix, the 10% target price is introduced by setting $E(\psi_1) = 0.10$ because the target price policy causes the output supply price to be greater than the output demand price. The planted acreage quantity restriction on input 1 is introduced by setting $E(\psi_2) = -0.10$. All other $E(\theta_i)$ entries in vector $\boldsymbol{b}$ are set to 0. The solution to (5.80) yields the following changes in the endogenous variables:

$$(5.81)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \\ E(\theta_{11}) \end{bmatrix} = \begin{bmatrix} 0.008 \\ -0.013 \\ 0.008 \\ 0.100 \\ -0.100 \\ 0.054 \\ 0.208 \\ 0.054 \\ -0.100 \\ 0.054 \\ -0.500 \\ 0.054 \\ 0.113 \\ 0.708 \end{bmatrix}.$$

The 10% target price coupled with a 10% reduction in planted acres increases the quantity produced, $E(q^S)$, and consumed, $E(q^D)$, by 0.8%. Consumer price, $E(p^D)$, declines by 1.3%, while the price received by producers, $E(p^S)$, increases by the mandated 10%. The target price policy creates a wedge between producer and consumer prices of 11.3% relative to the initial equilibrium price. The sum of the increase in producer price and the absolute value of the reduction in consumer price equals the shadow value, $E(\theta_{10})$, of 11.3%. This represents the subsidy that would be paid by the government on every bushel of grain produced if quantities were to clear the market.

The amount of planted acreage $E(x_1^D)$ declines by the legislated 10% reduction, while its derived demand value or price, $E(w_1^D)$, increases by 20.8%. The marginal cost or supply price, $E(w_1^S)$, decreases by 50% relative to its initial equilibrium price and the sum of the absolute values of these two price changes equals 70.8% (50% + 20.8%) of the initial equilibrium price, which equals the shadow value, $E(\theta_{11})$, as calculated by the EDM. The use of the substitute input, $E(x_2^D)$, increases by 5.4% and its demand price, $E(w_2^D)$, and supply price, $E(w_2^S)$, increase by 5.4%.

Equation (5.43) illustrated the impact of a target price policy that was 10% above the initial equilibrium price. In that example, the government would pay producers 21.1% more than they receive in the market for every bushel of grain produced. The target price policy would increase grain production by 6.7% relative to the initial equilibrium quantity. However, the simultaneously mandated reduction in planted acreage results in only a 0.8% increase in the amount of grain produced. In addition, the difference between the target price and the price producers received for grain, $E(\theta_{10}) = 11.3\%$, is lower than the difference that occurred without the input restriction (21.1%) relative to the original equilibrium price for all grain produced. Thus, coupling an acreage restriction with the target price reduces government expenditures relative to a target price program that did not include an acreage restriction.

Target Prices, Acreage Restrictions, and Yield Slippage

We conclude this discussion by adding an historical note. While planted-acreage set-aside programs were relatively easy to enforce, the U.S. government was not able to assess the quality of unplanted or, as termed, set-aside acreages. Producers had incentives to use their least productive land to meet the set-aside rules. Hence, while planted acreage declined by the legislated amount, grain production did not decline by the expected amount because of yield slippage. That is, setting aside lower-quality land did not reduce total production as much as had been anticipated. Coupled with the target price program, government expenditures on the program exceeded expectations. Anticipating the acreage quality/production quantity issue would have led to a more accurate assessment of the costs of the program. The actual outcome of the program could have been partially addressed by realizing that a 10% acreage reduction was not going to achieve the desired reduction in grain production.

One way to model this problem would be to assume that producers were likely to set aside marginal land. Hence, the actual bushel-basis reduction was likely to be similar to a reduction of planted acreage of only, say, 7.5%. To implement this example, we operationalize (5.80) by introducing the 10% target price, $E(\psi_1) = 0.10$, and simultaneously impose a 7.5% restriction on planted acreage, $E(\psi_2) = -0.075$. This smaller amount reflects yield or production slippage caused by setting aside relatively marginal land. After setting all other $E(\theta_i)$ entries in vector $\boldsymbol{b}$ to 0, the solution to (5.80) yields the following changes in the endogenous variables:

$$(5.82)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \\ E(\theta_{11}) \end{bmatrix} = \begin{bmatrix} 0.019 \\ -0.032 \\ 0.019 \\ 0.100 \\ -0.075 \\ 0.060 \\ 0.194 \\ 0.060 \\ -0.075 \\ 0.060 \\ -0.375 \\ 0.060 \\ 0.132 \\ 0.569 \end{bmatrix}.$$

The 10% target price, coupled with a 7.5% reduction in planted acreage, increases the quantity produced, $E(q^S)$, and consumed, $E(q^D)$, by 1.9%, more than double the 0.8% estimate in the previous example. Consumer price, $E(p^D)$, declines by 3.2%, while the price received by producers, $E(p^S)$, increases by the mandated 10%. The target price policy creates a wedge of 13.2% of the initial equilibrium price between producer and consumer prices. The sum of the increase in producer price and the absolute value of the reduction in consumer price is given by the shadow value, $E(\theta_{10})$, of 13.2%. Note that the subsidy paid by the government on every bushel of grain produced is larger in this example because yield slippage caused production to be larger than anticipated.

The amount of planted acreage, $E(x_1^D)$ and $E(x_1^S)$, declined by the legislated 7.5% amount, while its derived demand value or price, $E(w_1^D)$, increased by 19.4%. The marginal cost or supply price, $E(w_1^S)$, decreased by 37.5% relative to its initial equilibrium price and the sum of the absolute values of these two price changes is equal to 56.9%, which equals its shadow value, $E(\theta_{11})$, as calculated by the EDM. The use of input 2, $E(x_2^D)$ and $E(x_2^S)$, increased by 6% and its demand and supply price, $E(w_2^D)$ and $E(w_2^S)$, also increased by 6%.

This example illustrates how government outlays could have been more realistically estimated had the issue of yield slippage been considered. Alternatively, the policy could have increased the set-aside acreages beyond 10% to arrive at the desired reduction in grain production. An EDM could have been used to evaluate differing set-aside requirements and estimate government outlays.

Regulation of Relative Input Use

Concerns of environmental pollution or other negative externalities have led to the regulation of input use, with many regulations altering the relative amounts of inputs used to produce an output. For example, some European countries restrict the amount of nitrogen fertilizer that can be applied to an acre of land and many regions have restricted pesticide use. In the United States, Atwood and Helmers (1998) examine the effects of a proposed per acre fertilizer restriction on the quantity and quality of feed grain production. The restriction on the use of one input alters the relative amounts of inputs used to produce an output.

Assume that a policy mandates a reduction in the use of input 2 relative to input 1. For the case of two inputs, the policy can be modeled by defining

$$r = \frac{x_2}{x_1},$$

which is the ratio of amount of input 2 used relative to input 1. A policy that restricts the amount of input 2 that can be applied to an input such as land is a restriction on the size of r.

Figure 5.14 illustrates the effect of a relative input restriction. The relative input use restriction places related price wedges in both input markets but in opposite

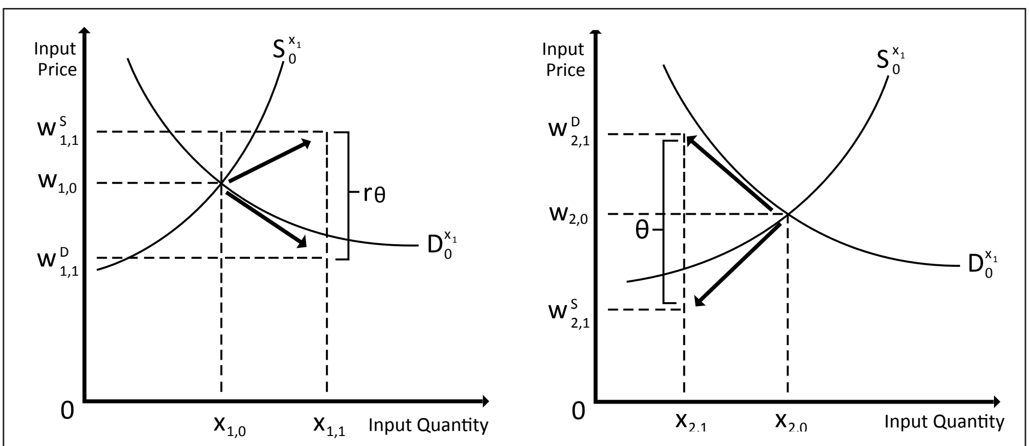

Figure 5.14. Regulation of Relative Input Use

directions. That is, the supply price of input 1, $w^S_{1,1}$, is greater than its demand price, $w^D_{1,1}$, while the demand price of input 2, $w^D_{2,1}$, is larger than its supply price, $w^S_{2,1}$. By construction, a regulation that limits the use of input 2 relative to input 1 necessarily increases the use of input 1 relative to input 2.

Figure 5.14 indicates that the effect of restricting the use of input 2 essentially provides a subsidy for input 1 that increases its use. Figure 5.14 illustrates that the resulting wedges between input prices are related. The wedge in the input 2 market equals θ and the wedge in the input 1 market equals $r\theta$. Note that at the original nonregulated equilibrium, the input demand and supply prices are equal ($\theta = 0$) and the original relative input use level is

$$r_0 = \frac{x_{2,0}}{x_{1,0}}.$$

Restricting the relative input use to

$$r_R = \frac{x_2}{x_1}$$

is equivalent to requiring

$$r_R x_1 - x_2 = 0. \tag{5.83}$$

To obtain the correct specification of the related wedges, we construct a Lagrangian of the producer's profit function with the relative input use restriction

$$\mathcal{L} = p^S f(\boldsymbol{x}) - w^S_1 x_1 - w^S_2 x_2 + \theta[r_R x_1 - x_2], \tag{5.84}$$

where w^S_i are the observed input prices.

Differentiating (5.84) with respect to x_1, x_2, and θ gives the following first-order conditions (FOCs):

$$p^S f_1 - w^S_1 + \theta r_R = 0 \tag{5.85}$$

$$p^S f_2 - w^S_2 - \theta = 0 \tag{5.86}$$

$$r_R x_1 - x_2 = 0 \tag{5.87}$$

where w^S_1 and w^S_2 are the prices that producers pay for inputs 1 and 2, respectively. However, (5.85) and (5.86) indicate that the constraint on input usage causes producers to behave "as if" their input demand prices were $w^D_1 = w^S_1 - \theta r_R$ and $w^D_2 = w^S_2 + \theta$. Therefore, the following system of equations models producer decision making "as if" input demand prices were

(5.88) $p^S f_1 - w_1^D = 0$

(5.89) $p^S f_2 - w_2^D = 0$

(5.90) $w_1^D = w_1^S - r_R\theta$

(5.91) $w_2^D = w_2^S + \theta$

(5.92) $x_2 = r_R x_1.$

The resulting system indicates that the relative input use restriction has driven wedges between input prices, as demonstrated in Figure 5.14, with the net effect being equivalent to simultaneously providing a per unit subsidy of $r_R\theta$ in the input 1 market and imposing a per unit tax of θ in the input 2 market. Consequently, the resulting value of θ is endogenous.

To convert (5.90)–(5.92) into a proportional elasticity form, we begin at the original unregulated equilibrium, with

$$r_0 = \frac{x_{2,0}}{x_{1,0}}$$

and $w_{i,0}^D = w_{i,0}^S$ or $\theta_0 = 0$. Taking the total differential of (5.90) yields

(5.93) $dw_1^D = dw_1^S - r_0 d\theta - \theta_0 dr_R.$

At the initial equilibrium, (5.93) is

(5.94) $dw_1^D = dw_1^S - \frac{x_{2,0}}{x_{1,0}} d\theta - 0dr_R.$

Multiplying (5.94) by $\frac{1}{w_{1,0}^D}$ results in

(5.95) $\frac{dw_1^D}{w_{1,0}^D} = \frac{dw_1^S}{w_{1,0}^D} - \frac{x_{2,0}}{x_{1,0}} \frac{1}{w_{1,0}^D} d\theta.$

Multiplying the first term on the right-hand side of (5.95) by $\frac{w_{1,0}^S}{w_{1,0}^S}$ and the second term by $\frac{w_{2,0}^S}{w_{2,0}^S}$ results in

(5.96) $\frac{dw_1^D}{w_{1,0}^D} = \frac{dw_1^S}{w_{1,0}^D} \frac{w_{1,0}^S}{w_{1,0}^S} - \frac{x_{2,0}}{x_{1,0}} \frac{w_{2,0}^S}{w_{2,0}^S} \frac{1}{w_{1,0}^D} d\theta,$

or, upon rearranging terms,

(5.97) $\frac{dw_1^D}{w_{1,0}^D} = \frac{w_{1,0}^S}{w_{1,0}^D} \frac{dw_1^S}{w_{1,0}^S} - \frac{w_{2,0}^S}{w_{1,0}^D} \frac{x_{2,0}}{x_{1,0}} \frac{d\theta}{w_{2,0}^S}.$

At the initial equilibrium point, $w_{1,0}^D = w_{1,0}^S$, (5.97) is

(5.98) $$\frac{dw_1^D}{w_{1,0}^D} = \frac{dw_1^S}{w_{1,0}^S} - \frac{w_{2,0}^S x_{2,0}}{w_{1,0}^D x_{1,0}} \frac{d\theta}{w_{2,0}^S}$$

or, in proportional elasticity form,

(5.99) $$E(w_1^D) = E(w_1^S) - \frac{K_2}{K_1} E(\theta_{12}),$$

where K_1 and K_2 are factor shares.

The total differential of (5.91) gives

(5.100) $$dw_2^D = dw_2^S + d\theta.$$

Multiplying (5.100) by $\frac{1}{w_{2,0}^D}$ results in

(5.101) $$\frac{dw_2^D}{w_{2,0}^D} = \frac{dw_2^S}{w_{2,0}^D} - \frac{1}{w_{2,0}^D} d\theta.$$

Multiplying each term on the right-hand side of (5.101) by $\frac{w_{2,0}^S}{w_{2,0}^S}$ results in

(5.102) $$\frac{dw_2^D}{w_{2,0}^D} = \frac{w_{2,0}^S}{w_{2,0}^D} \frac{dw_2^S}{w_{2,0}^S} + \frac{w_{2,0}^S}{w_{2,0}^D} \frac{d\theta}{w_{2,0}^S}.$$

Because $w_{2,0}^D = w_{2,0}^S$ at the initial equilibrium point, (5.102) can be written in proportional elasticity form as

(5.103) $$E(w_2^D) = E(w_2^S) + E(\theta_{12}).$$

To complete the model, the total differential of (5.92) is given by

(5.104) $$dx_2 = r_0 dx_1 + x_{1,0} dr_R.$$

Multiplying (5.104) by $\frac{1}{x_{2,0}}$ yields

(5.105) $$\frac{dx_2}{x_{2,0}} = \frac{1}{x_{2,0}} \frac{x_{2,0}}{x_{1,0}} dx_1 + \frac{x_{1,0}}{x_{2,0}} dr_R$$

or

(5.106) $$\frac{dx_2}{x_{2,0}} = \frac{dx_1}{x_{1,0}} + \frac{dr_R}{r_0}.$$

In proportional elasticity form, (5.106) becomes

(5.107) $E(x_2) = E(x_1) + E(r_R).$

The EDM for this problem requires that $E(\theta_{12})$ in (5.99) and (5.103) be an endogenous variable. Therefore, we move $E(\theta_{12})$ to the left-hand side of each equation such that

(5.108) $E(w_1^D) - E(w_1^S) + \frac{K_2}{K_1}E(\theta_{12}) = 0$

and

(5.109) $E(w_2^D) - E(w_2^S) - E(\theta_{12}) = 0.$

Equations (5.108) and (5.109) replace (5.28) and (5.29) in the EDM. In addition, the following restriction requires that $E(x_1)$ be moved to the left-hand side of (5.107) to operationalize the model as

(5.110) $E(x_2) - E(x_1) = E(r_R).$

Therefore, the EDM for the relative input usage problem consists of (5.18)–(5.27), (5.108), (5.109), and (5.110) which results in

(5.111)
$$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & K_2/K_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ 0 \\ 0 \\ E(r_R) \end{bmatrix}.$$

A policy that reduces the use of input 2 by 10% relative to input 1 is evaluated in the EDM by setting $E(r_R)$ equal to –0.10 in vector $\boldsymbol{b}$, with all other elements set to 0. The solution to (5.111) is

$$(5.112)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{12}) \end{bmatrix} = \begin{bmatrix} -0.022 \\ 0.036 \\ -0.022 \\ 0.036 \\ 0.048 \\ -0.052 \\ -0.034 \\ 0.066 \\ 0.048 \\ -0.052 \\ 0.241 \\ -0.052 \\ 0.118 \end{bmatrix}.$$

The EDM solution indicates that the use of input 1 increases by 4.8% and the use of input 2 declines by 5.2%. Note that the sum of the absolute values of the two changes is equal to the percentage change as dictated by the policy, $E(x_2) - E(x_1) = -0.10$. The percentage change in relative input use,

$$\frac{1+E(x_2)}{1+E(x_1)} = \frac{0.948}{1.048} = 0.0905$$

differs slightly from the expected value of 0.090 because of the EDM's linear approximation of the system.[7] With the relative input restriction, the output quantity, $E(q^S)$, decreases by 2.2% and output price, $E(p^S)$, increases by 3.6%. The market price of input 1, $E(w_1^S)$, increases by 24.1% and the market price of input 2, $E(w_2^S)$, decreases by 5.2%.

Summary

This chapter expands the one-output, two-input model presented in Chapter 4 to allow for the modeling of policy or regulatory shocks. Policy-related actions are modeled within an EDM framework as price or quantity wedges added to the relevant equilibrium equations. Initial wedges may be predetermined by a specific legislative action, but the effects on other variables are endogenous to the system.

Some policy actions result in wedges between demand and supply prices or quantities that are themselves endogenous to the system. To accommodate this endogeneity, policy and regulatory actions must be modeled with one or more additional restrictions. The process is analogous to the use of Lagrangian equa-

7 Using a smaller regulatory ratio shock of –0.01 on the right-hand side of (5.111) for $E(r_R)$ results in $\frac{1+E(x_2)}{1+E(x_1)} = 0.99$, indicating that the EDM-estimated factor ratio declines by an amount quite close to 1%.

tions as means for adding a constraint to an optimization process. The addition of each restriction requires that a new parameter—a shadow value—be added to the system for the purpose of system identification. The restriction that is added is specific to the issue being addressed. It is often useful to graphically illustrate the effects of policies on supply and demand functions prior to specifying changes in EDM equilibrium equations or adding new restrictions.

»Chapter Six

RELAXING THE ASSUMPTION OF PERFECT COMPETITION

To this point, we have developed and implemented equilibrium displacement models (EDMs) assuming that markets are perfectly competitive (i.e., individual producers cannot influence prices received or paid). In this chapter, we demonstrate how EDMs can be used to model market structures that vary from a single-seller monopoly to monopolistic competition. We define market power as a situation in which prices paid or received for a good or service differ from their marginal costs of production.[8] EDMs can be used to model a monopolistically competitive environment in which individual firms have a small degree of market power for their own products while facing a more inelastic own-price elasticity of demand in the aggregate product market. This type of structure is common in the food sector and often occurs in the form of brand name recognition.

Market Power in the Output Market

Given an EDM's ability to accommodate differences between the prices paid by consumers, p^D, and the per unit marginal cost of production (i.e., producer price), p^S, we demonstrate how an EDM can be used to model market power. We use price wedges to show that market power can be modeled as if consumers are being charged a per unit tax by a monopolist. Similarly, monopsony market power could be modeled as a "tax" on input purchases. We present a model in which a monopolist initially determines and sets its profit maximizing output and selling price, p^D, given the own-price elasticity of demand for its product. Further, we consider the impact of regulatory actions on a monopolist by placing a ceiling on its output price to move monopoly prices toward competitive equilibrium prices.

8 Monopsony power in input markets can be similarly defined using wedges between marginal values of and prices paid for inputs.

A Model of Market Power

If an output market is imperfectly competitive, the result is higher output prices and/or lower input prices and reduced industry output. Consequently, the mechanism used to model the impacts of market power on price and quantity equilibria is analogous to that used to estimate changes in equilibria caused by a tax on output. These effects can be estimated by placing a wedge between consumer and producer prices in the output market. However, the interpretation of changes in equilibrium prices differs from the tax example.

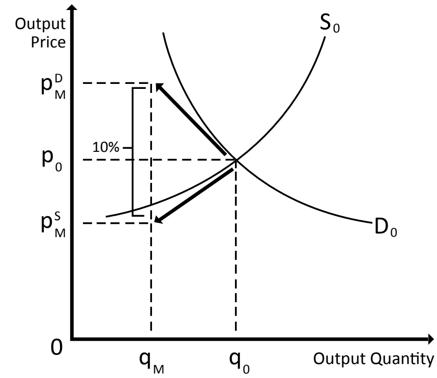

Figure 6.1. Imperfect Competition in the Output Market

If market power is assumed to cause a 10% price wedge between output demand and supply prices, then Figure 6.1 illustrates the impact of an imperfectly competitive market.[9] In a perfectly competitive market, the equilibrium price would be p_0 and the equilibrium quantity would be q_0. Imperfect competition places a price wedge between the price paid by consumers, p_M^D, and the per unit costs incurred by producers, p_M^S. In addition, the equilibrium quantity exchanged in the market, q_M, is reduced from the initial equilibrium quantity, q_0.

The 10% price wedge would be entered in vector $\boldsymbol{b}$ of (5.31) as $E(\theta_{10}) = 0.10$ with all other values set equal to 0. The change in endogenous variables are obtained by solving (5.32):

$$(6.1) \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \end{bmatrix} = \begin{bmatrix} -0.032 \\ 0.053 \\ -0.032 \\ -0.047 \\ -0.013 \\ -0.039 \\ -0.066 \\ -0.039 \\ -0.013 \\ -0.039 \\ -0.066 \\ -0.039 \end{bmatrix}.$$

9 We use a 10% wedge as an example. Little empirical work exists to indicate whether such a measure is representative for any particular industry. For output markets, the wedge represents the difference between consumer prices and the marginal costs of production, which are difficult to measure or estimate. New empirical industrial organization techniques often estimate a Lerner index using own-price elasticities of demand as a proxy for this wedge (Bhuyan and Lopez, 1997).

Similar to the effects of a 10% tax, market power in this industry causes a 3.2% decrease in quantity of output produced, $E(q^S)$, and consumed, $E(q^D)$, and consumers pay 5.3% more than the original equilibrium price, $E(p^D)$. However, assuming that market power occurs in the processing sector, producers in that sector receive the consumer price for their output and incur a 4.7% reduction in their marginal costs of production, $E(p^S)$, relative to the original equilibrium price. Note that the sum of the absolute values of these two changes represents the

From: Student@UEconomics.edu
To: Professor Watson
Date: Thursday, 11 Nov 2021 at 12:01 p.m.
Subject: Chapter 6

Dear Professor Watson,

I am a little confused when you refer to market power and monopoly. Aren't those against the law and, if they are hard to parameterize, then why are we doing them?

Ning

From: Professor Watson
To: Student@UEconomics.edu
Date: Thursday, 11 Nov 2021 at 14:08 p.m.
Re: Chapter 6

Dear Class,

This is a good question, Ning. We spoke about monopolistic or oligopolistic competition because many industries in the food sector exhibit this form of market structure. Namely, firms try to create differentiated products and services to acquire buyer loyalty. Essentially, this is an attempt to make the demand curve for a firm's product more inelastic. There is nothing wrong with these efforts because buyers value product differentiation. I used the example of product branding in class. Not all brands are equal, right? Consider Interbrand's annual list of the 100 most valuable brands, which can be found at https://interbrand.com/best-brands/. There are a number of well-known food brands on that list. Those of you who have taken Professor Wang's food marketing course have likely studied determinants of such brands.

It is very difficult to create parameters that measure market power. The exercise of market power can, however, harm consumers and producers if price manipulation occurs. And collusion among firms to set market quantities and prices is illegal. However, we must recognize that many industries in agriculture have only a few buyers for farm-gate commodities or that concentration exists in processing industries that supply restaurants, hospital, and other institutions (including retail grocery outlets). Thus, an EDM model can be created to recognize the structure of an industry.

All the best,
Dr. Watson

assumed 10% wedge. In this example, market power reduces the use of input 1 by 1.3% with a concurrent decline of 6.6% in price. The use, $E(x_2^D)$, and price, $E(w_2^D)$, of input 2 decline by 3.9%. Finally, note that the price consumers pay for the reduced level of output is greater than the supply or marginal costs of producing the output, as illustrated in Figure 6.1. This difference, coupled with the reduction in output, represents the basis for deadweight losses that result from imperfect market competition.

Monopoly Pricing and a Price Ceiling

The previous example assumed a known price wedge between the price consumers pay for output and the per unit cost of producing that output that results from market power. If we assume an unregulated monopoly, then the size of a monopolist's profit-maximizing price wedge is a well-known function of the own-price elasticity of demand for its product.

To develop an EDM that incorporates this issue, we begin with the monopolist's profit function,

$$(6.2)\quad \pi = p_M^D(q(\boldsymbol{x}))q(\boldsymbol{x}) - \boldsymbol{w^D x},$$

and differentiate it with respect to $\boldsymbol{x}$ to obtain the following first-order conditions:

$$(6.3)\quad \boldsymbol{\pi_x} = \frac{dp_M^D}{dq}\boldsymbol{f_x}q + p_M^D\boldsymbol{f_x} - \boldsymbol{w^D} = 0$$

or

$$(6.4)\quad \boldsymbol{\pi_x} = \left(p_M^D + \frac{dp_M^D}{dq}q\right)\boldsymbol{f_x} - \boldsymbol{w^D} = 0.$$

If we define

$$(6.5)\quad p_M^S = p_M^D + \frac{dp_M^D}{dq}q,$$

then we can write the monopolist's FOCs as

$$(6.6)\quad p_M^S\boldsymbol{f_x} - \boldsymbol{w^D} = 0$$

to obtain an expression that provides the monopoly relationship between per unit cost, p_M^S, and the price consumers pay for the monopoly output, p_M^D, as noted in (6.5). Before taking total differentials and constructing proportional elasticity equations, we multiply the second term of the right-hand side of (6.5) by $\frac{p_M^D}{p_M^D}$:

(6.7) $p_M^S = p_M^D + \frac{dp_M^D}{dq}\frac{q}{p_M^D}p_M^D$

or

(6.8) $p_M^S = p_M^D\left(1 + \frac{1}{\eta^D}\right),$

where η^D is the own-price elasticity of demand at the monopolist's optimally chosen output level, q_M. The well-known result is that a monopolist's optimal output level occurs on the elastic portion of the demand curve, where $|\eta^D| > 1$. We assume that a monopolist chooses their initial unregulated output level, q_M, and the resulting prices, p_M^D and p_M^S, as indicated in (6.8) and in Figure 6.2.

If we divide (6.8) by p_M^D, we obtain the market power initial equilibrium price ratio (R_{SD}^M), used in the following EDM derivations:

(6.9) $R_{SD}^M = \frac{p_M^S}{p_M^D} = (1 + \frac{1}{\eta^D}).$

Assume a monopolist faces a regulatory price ceiling set at level $p_R^D < p_M^D$, or

(6.10) $p_R^D = p_M^D + \psi_1,$

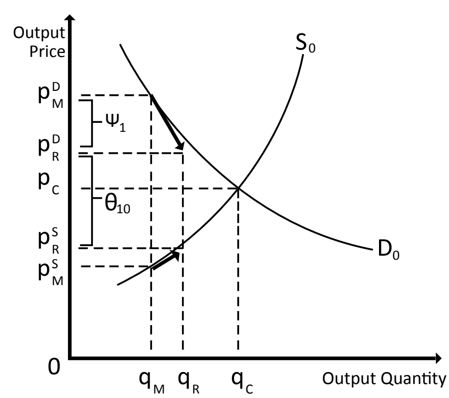

Figure 6.2. A Regulated Monopoly

where ψ_1 is an exogenously mandated reduction in consumer price for the output. To model the after-regulation price wedge, let θ_{10} be an endogenous variable:

(6.11) $p^D = p^S + \theta_{10}.$

Taking the total differential of (6.11) results in

(6.12) $dp^D = dp^S + d\theta_{10}.$

Multiplying (6.12) by $\frac{1}{p_M^D}$ results in

(6.13) $\frac{dp^D}{p_M^D} = \frac{dp^S}{p_M^D} + \frac{d\theta}{p_M^D}.$

Multiplying the first term on the left-hand side of (6.13) by $\frac{p_M^S}{p_M^S}$ yields

(6.14) $\frac{dp^D}{p_M^D} = \frac{dp^S}{p_M^D}\frac{p_M^S}{p_M^S} + \frac{d\theta_{10}}{p_M^D}$

or, using (6.9),

$$\frac{dp^D}{p_M^D} = R_{SD}^M \frac{dp^S}{p_M^S} + \frac{d\theta_{10}}{p_M^D}. \tag{6.15}$$

In proportional elasticity form, (6.15) becomes

$$E(p_M^D) = R_{SD}^M E(p_M^S) + E(\theta_{10}). \tag{6.16}$$

The necessary restriction to impose a regulatory price ceiling is obtained by differentiating (6.10) given the initial monopoly price, which yields

$$dp^D = d\psi_1. \tag{6.17}$$

Multiplying (6.17) by $\frac{1}{p_M^D}$ results in

$$\frac{dp^D}{p_M^D} = \frac{d\psi}{p_M^D} \tag{6.18}$$

or

$$E(p^D) = E(\psi_1), \tag{6.19}$$

with $E(\psi_1) < 0$. After moving the endogenous variables to the left-hand side in (6.16), the EDM for this problem consists of (5.18)–(5.26), (6.16), (5.28), (5.29), and (6.19). With these changes, (5.31) becomes

$$\begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -R_{SD}^M & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ 0 \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\psi_1) \end{bmatrix} \tag{6.20}$$

We assume that $\eta^D = -2.0$, which results in

(6.21) $R_{SD}^{M} = \left(1 + \frac{1}{\eta^{D}}\right) = (1 + \frac{1}{-2}) = 0.50.$[10]

Setting $E(\psi_1) = -0.10$ and all other $E(\theta_i) = 0$ in vector $\boldsymbol{b}$, the solution to (6.20) is

$$(6.22)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(\theta_{10}) \end{bmatrix} = \begin{bmatrix} 0.200 \\ -0.100 \\ 0.200 \\ 0.300 \\ 0.083 \\ 0.250 \\ 0.417 \\ 0.250 \\ 0.083 \\ 0.250 \\ 0.417 \\ 0.250 \\ 0.250 \end{bmatrix}.$$

The results in (6.22) indicate that the regulatory policy reduces consumer prices, $E(p^D)$, by the expected 10% while monopoly output, $E(q^D)$ and $E(q^S)$, increases by 20%. The per unit cost, $E(p^S)$, at the regulatory-induced output level increases by 30%. The increased output increases both input quantities and prices. The shadow value, $E(\theta_{10})$, equals 25%, which is the difference between the after-regulation demand and supply prices as a proportion of the monopoly original demand price, p_M^D.

To verify this result, we note that

$$(6.23)\quad p^D = p_M^D\left(1 + E(p^D)\right)$$

and

$$(6.24)\quad p^S = p_M^S\left(1 + E(p^S)\right) = R_{SD}^{M} p_M^D (1 + E(p^S)).$$

Subtracting (6.24) from (6.23) results in the Lerner index of

$$(6.25)\quad \frac{p^D - p^S}{p_M^D} = \left(1 + E(p^D)\right) - R_{SD}^{D}\left(1 + E(p^S)\right).$$

Using the results presented in (6.22), expression (6.25) results in

10 It is common to choose numbers that might make sense theoretically in these types of models because actual values are difficult to estimate empirically.

(6.26) $\frac{p^D - p^S}{p_M^D} = (1 - 0.10) - \left(0.5(1 + 0.30)\right) = 0.90 - 0.65 = 0.25.$

The implication of (6.26) is that imperfect competition causes the price that consumers pay for the good or service to be 25% higher than the marginal cost of producing the good or service.

Price Regulation Needed to Obtain the Competitive Equilibrium Result

The inverse of the **A** matrix presented in (6.20) is used to estimate the amount of the price reduction, $E(\psi_1)$, that would be required to generate a competitive price outcome such that $p^D = p^S$. To identify this value, we use the multiplier values m_D and m_S from the second ($m_D = 1.00$) and fourth ($m_S = -3.00$) rows of the last column of the $\boldsymbol{A}^{-1}$ matrix presented in the accompanying Excel workbook.

The system implies that

(6.27) $p^D = p_M^D(1 + m_D\psi_1)$

and

(6.28) $p^S = p_M^S(1 + m_S\psi_1).$

Substituting (6.9) into (6.28) for p_M^S yields

(6.29) $p^S = R_{SD}^M p_M^D(1 + m_S\psi_1).$

Dividing (6.29) by (6.27) yields

(6.30) $\frac{p^S}{p^D} = R_{SD}^M\left(\frac{1 + m_S\psi_1}{1 + m_D\psi_1}\right).$

The competitive price result is given by $\frac{p^S}{p^D} = 1$. Therefore, in the case of a competitive equilibrium outcome, (6.30) would be

(6.31) $1 = R_{SD}^M\left(\frac{1 + m_S\psi_1}{1 + m_D\psi_1}\right).$

Multiplying both sides of (6.31) by $1 + m_D\psi_1$ results in

(6.32) $(1 + m_D\psi_1) = R_{SD}^M\ (1 + m_S\psi_1).$

Expanding (6.32) yields

(6.33) $1 + m_D\psi_1 = R_{SD}^M + R_{SD}^M m_S\psi_1$

or

(6.34) $(R_{SD}^{M} m_S \psi_1 - m_D \psi_1) = 1 - R_{SD}^{M}$

and, upon solving for ψ_1,

(6.35) $(R_{SD}^{M} m_S - m_D)\psi_1 = (1 - R_{SD}^{M})$

or

(6.36) $\psi_1 = \frac{1 - R_{SD}^{M}}{R_{SD}^{M} m_S - m_D}.$

Using our numerical example for which $R_{SD}^{M} = 0.5$, $m_D = 1.00$, and $m_S = -3.0$, (6.36) becomes

(6.37) $\psi_1 = \frac{(1-0.5)}{(0.5(-3)-1)} = -0.20.$

Therefore, 0.20 (20%) represents the size of the regulatory price ceiling that reduces consumer price to its competitive equilibrium. That is, placing a price ceiling on the good or service that is 20% below the imperfectly competitive price would generate a competitive price and quantity equilibrium. As a check on this value, note that using (6.30) yields

(6.38) $\frac{P^S}{P^D} = R_{SD}^{M}\left(\frac{1+m_S\psi_1}{1+m_D\psi_1}\right) = 0.5\left(\frac{(1-3(-0\ 20))}{(1+1(-0.20))}\right) = 0.5\left(\frac{1.6}{0.8}\right) = 1.0.$

Monopolistic Competition

The perfectly competitive model assumes that individual firms face infinitely elastic demand curves while equilibria at the industry level could occur in the elastic or inelastic range of the aggregate output demand curve. For monopolistic competition, we relax the perfectly competitive assumption and allow firms to have a small amount of market power. That is, an industry may function in an area of the market demand curve that is in the elastic or inelastic range. However, firms within that industry face a more elastic demand curve for their products relative to the overall market demand curve. This is common for monopolistically competitive firms for which brand recognition provides some small amount of market power. While a brand allows a producer to trade off quantity and price to some degree, the size of this trade-off is substantially influenced by the number and quality of substitutes for each brand and the market demand elasticity.

Assume that each firm within an industry faces an own-price elasticity of demand for their branded product of $\eta^f = -20.0$ and that the own-price elasticity of demand for all products in the industry is $\eta^D = -0.60$. In this case, the initial price wedge in (6.9) uses the firm-level elasticity of demand:[11]

$$(6.39) \quad R_{SD}^{MC} = \frac{P_{MC}^S}{P_{MC}^D} = \left(1 + \frac{1}{\eta^f}\right) = \left(1 + \frac{1}{-20.0}\right) = 0.95.$$

In our general EDM system, if we replace (5.27) with

$$(6.40) \quad E(p^D) = R_{SD}^{MC} E(p^S) + E(\theta_{10}),$$

we can use the model to consider monopolistically competitive responses to policy changes.

Consequently, we have two ways to incorporate imperfect competition in EDMs. One approach is to place a "tax" between demand and supply prices in a market that represents the impact of imperfect competition. Alternatively, we can directly include a Lerner index that represents market power in an EDM. The two approaches yield similar results.

Summary

This chapter demonstrated that EDMs can be used to relax the assumption of perfectly competitive input or output markets. Imperfect competition places a wedge between the prices that consumers pay for a product or service and its marginal cost of production. Hence, imperfect competition can be modeled by EDMs as if a tax has been placed on a good or service. Further, many economic sectors are characterized by monopolistic competition. By including a Lerner index, EDMs can also be used to model those situations.

11 Note that as $n^f \to -\infty$, $R_{SD}^{MC} \to 1$ and we obtain the original competitive result in expression (5.15) as $E(\theta_{10}) \to 0$.

»Chapter Seven

EDM POLICY APPLICATIONS: MULTIPLE MARKETS

In Chapters 5 and 6, we considered an equilibrium displacement model (EDM) for a one-output, two-input production process that occurred at a single market stage. In this chapter, we expand that basic model to include three inputs and consider both horizontal and vertical linkages in output markets. We demonstrate that EDMs can accommodate a wide variety of market linkages. For example, EDMs can be constructed to consider both imports and exports or to incorporate product substitutes and vertically connected market stages.

Product Imports and Exports

The following example represents a model of the U.S. beef industry. The U.S. beef production sector uses cattle and other inputs to produce consumer beef products. In addition, processing and fabrication companies both import and export beef. Some processing firms operate across global markets. We assume that the supply of imported beef, q_I^S, is a function of the price of imported beef, p^I, and that the demand for exported beef, q_E^D, is a function of the price of exported beef, p^E. Therefore,

$$(7.1) \quad q_I^S = q_I^S(p^I);$$

$$(7.2) \quad q_E^D = q_E^D(p^E).$$

Equation (7.1) represents the supply of imported beef offered to the U.S. market by foreign suppliers. When the price of beef increases, foreign suppliers of beef in countries such as Australia, New Zealand, Canada, and Mexico are willing to increase their quantity supplied of beef to the U.S. market. Given this upward-sloping supply function, decreases in the price that foreign producers receive for beef in the United States reduce the quantity supplied of imported beef. Equation (7.2) represents the foreign demand for U.S. beef exports. When the price of U.S. beef that is exported to other countries increases, consumers in foreign countries reduce their quantity demanded. The opposite scenario is also true. Converting (7.1) and (7.2) into proportional elasticity forms under the assumption that quantities of imports and exports demanded are equal to the quantity of imports and exports supplied, we obtain

$$(7.3) \quad E(q_I) = \varepsilon_I E(p^I)$$

$$(7.4) \quad E(q_E) = \eta_E E(p^E)$$

where ε_I is the own-price elasticity of supply of imported beef and η_E is the own-price elasticity of demand for beef exports.

The amount of beef available in the domestic market, q^D, is a combination of domestic production, q^S, plus the quantity of imports, q_I, less the quantity of exports, q_E, which is illustrated by

$$(7.5) \quad q^D = q^S + q_I - q_E.$$

Totally differentiating (7.5) yields:

$$(7.6) \quad dq^D = dq^S + dq_I - dq_E.$$

Dividing (7.6) by q^D results in

$$(7.7) \quad \frac{dq^D}{q^D} = \frac{dq^S}{q^D} + \frac{dq_I}{q^D} - \frac{dq_E}{q^D}.$$

To convert to proportional elasticities, the first term on the right-hand side of (7.7) is multiplied by $\frac{q^S}{q^S}$, the second term by $\frac{q_I}{q_I}$, and the third term by $\frac{q_E}{q_E}$:

$$(7.8) \quad \frac{dq^D}{q^D} = \left(\frac{q^S}{q^D}\right)\frac{dq^S}{q^S} + \left(\frac{q_I}{q^D}\right)\frac{dq_I}{q_I} - \left(\frac{q_E}{q^D}\right)\frac{dq_E}{q_E},$$

or

$$(7.9) \quad E(q^D) = \mathcal{R}_S E(q^S) + \mathcal{R}_I E(q_I) - \mathcal{R}_E E(q_E),$$

where $\mathcal{R}_S$ represents a proportional quantity weight of domestic production, $\frac{q^S}{q^D}$, $\mathcal{R}_I$ represents the proportional quantity weight of imports, $\frac{q_I}{q^D}$, and $\mathcal{R}_E$ represents the proportional quantity weight of exports, $\frac{q_E}{q^D}$. Note that in the absence of storage, $\mathcal{R}_s + \mathcal{R}_I - \mathcal{R}_E$ must sum to 1 (or 100% in percentage terms). For example, since 2010, the United States has annually exported about 8% of its total beef supply, $\mathcal{R}_E$, and imported about 12% of its total beef supply, $\mathcal{R}_I$. Consequently, $\mathcal{R}_S$ = 96% (96% + 12% – 8% = 100%).

For simplicity of illustration, U.S. beef imports and exports are assumed to be of similar quality. However, EDMs can easily accommodate product differentiation by using differing prices. Two additional equilibrium equations are needed to identify the expanded system. In the absence of market interventions and under the assumption of similar quality, we assume that the price of imported beef, p^I, and the price of exported beef, p^E, are both equal to the price of domestic beef, p^D, so that

(7.10) $$p^I = p^D$$

(7.11) $$p^E = p^D.$$

To convert to proportional elasticity forms, we totally differentiate (7.10) and (7.11) to yield

(7.12) $$dp^I = dp^D$$

(7.13) $$dp^E = dp^D.$$

Dividing (7.12) by p^I and (7.13) by p^E results in

(7.14) $$\frac{dp^I}{p^I} = \frac{dp^D}{p^I}$$

(7.15) $$\frac{dp^E}{p^E} = \frac{dp^D}{p^E}.$$

Multiplying the right-hand side of equations (7.14) and (7.15) by $\frac{p^D}{p^D}$ yields

(7.16) $$\frac{dp^I}{p^I} = \left(\frac{p^D}{p^I}\right)\frac{dp^D}{p^D}$$

(7.17) $$\frac{dp^E}{p^E} = \left(\frac{p^D}{p^E}\right)\frac{dp^D}{p^D}$$

and converting to proportional elasticity forms

(7.18) $$E(p^I) = \left(\frac{p^D}{p^I}\right)E(p^D)$$

(7.19) $$E(p^E) = \left(\frac{p^D}{p^E}\right)E(p^D).$$

Note that $\frac{p^D}{p^I}$ and $\frac{p^D}{p^E}$ are equal to 1 if price wedges between domestic beef prices and import and export prices do not exist. These values can be altered to reflect differences in quality of imports and exports.

An EDM that includes imports and exports is developed using our base model from Chapter 5, which includes (5.18)–(5.23) and (5.25)–(5.29), and adding the terms representing horizontal linkages using (7.3), (7.4), (7.9), (7.18), and (7.19). This results in

(7.20) $E(q^D) = \eta^D E(p^D) + E(\theta_1)$

(7.21) $E(p^S) = K_1 E(w_1^D) + K_2 E(w_2^D) + E(\theta_2)$

(7.22) $E(x_1^D) = E(q^S) + K_1\sigma_{11} E(w_1^D) + K_2\sigma_{12} E(w_2^D) + E(\theta_3)$

(7.23) $E(x_2^D) = E(q^S) + K_1\sigma_{21} E(w_1^D) + K_2\sigma_{22} E(w_2^D) + E(\theta_4)$

(7.24) $E(x_1^S) = \varepsilon_1 E(w_1^S) + E(\theta_5)$

(7.25) $E(x_2^S) = \varepsilon_2 E(w_2^S) + E(\theta_6)$

(7.26) $E(q_I) = \varepsilon_I E(p^I) + E(\theta_7)$

(7.27) $E(q_E) = \eta_E E(p^E) + E(\theta_8)$

(7.28) $E(q^d) = \mathcal{R}_s E(q^s) + \mathcal{R}_I E(q_I) - \mathcal{R}_E E(q_E)$

(7.29) $E(x_1^D) = E(x_1^S) + E(\theta_9)$

(7.30) $E(x_2^D) = E(x_2^S) + E(\theta_{10})$

(7.31) $E(p^D) = E(p^S) + E(\theta_{11})$

(7.32) $E(w_1^D) = E(w_1^S) + E(\theta_{12})$

(7.33) $E(w_2^D) = E(w_2^S) + E(\theta_{13})$

(7.34) $E(p^I) = \left(\frac{p^D}{p^I}\right) E(p^D) + E(\theta_{14})$

(7.35) $E(p^E) = \left(\frac{p^D}{p^E}\right) E(p^D) + E(\theta_{15}).$

Moving the endogenous variables of (7.20)–(7.35) to the left-hand side yields

(7.36) $E(q^D) - \eta^D E(p^D) = E(\theta_1)$

(7.37) $E(p^S) - K_1 E(w_1^D) - K_2 E(w_2^D) = E(\theta_2)$

(7.38) $E(x_1^D) - E(q^S) - K_1 \sigma_{11} E(w_1^D) - K_2 \sigma_{12} E(w_2^D) = E(\theta_3)$

(7.39) $E(x_2^D) - E(q^S) - K_1 \sigma_{21} E(w_1^D) - K_2 \sigma_{22} E(w_2^D) = E(\theta_4)$

(7.40) $E(x_1^S) - \varepsilon_1 E(w_1^S) = E(\theta_5)$

(7.41) $E(x_2^S) - \varepsilon_2 E(w_2^S) = E(\theta_6)$

(7.42) $E(q_I) - \varepsilon_I E(p^I) = E(\theta_7)$

(7.43) $E(q_E) - \eta_E E(p^E) = E(\theta_8)$

(7.44) $E(q^d) - \mathcal{R}_s E(q^s) - \mathcal{R}_I E(q_I) + \mathcal{R}_E E(q_E) = 0$

(7.45) $E(x_1^D) - E(x_1^S) = E(\theta_9)$

(7.46) $E(x_2^D) - E(x_2^S) = E(\theta_{10})$

(7.47) $E(p^D) - E(p^S) = E(\theta_{11})$

(7.48) $E(w_1^D) - E(w_1^S) = E(\theta_{12})$

(7.49) $E(w_2^D) - E(w_2^S) = E(\theta_{13})$

(7.50) $E(p^I) - \left(\frac{p^D}{p^I}\right) E(p^D) = E(\theta_{14})$

(7.51) $E(p^E) - \left(\frac{p^D}{p^E}\right) E(p^D) = E(\theta_{15})$.

The exogenous shocks represented by θ_1–θ_{15} can be used to evaluate policies or interventions to the system of equations. The choice of shock depends upon the specific application. Using linear algebra, (7.36)–(7.51) can be written as

(7.52) $\boldsymbol{Ay} = \boldsymbol{b}$, where $\boldsymbol{A}$ is a 16×16 matrix of parameters, $\boldsymbol{y}$ is a 16×1 vector of endogenous variables, and $\boldsymbol{b}$ is a 16×1 vector of exogenous shocks such that

$$
(7.53)\quad
\begin{bmatrix}
1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\varepsilon_I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\eta_E \\
1 & 0 & -\mathcal{R}_S & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{R}_I & 0 & \mathcal{R}_E & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & -p^D/p^I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -p^D/p^E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_1^S) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(q_I) \\ E(p^I) \\ E(q_E) \\ E(p^E)
\end{bmatrix}
=
\begin{bmatrix}
E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ 0 \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\theta_{13}) \\ E(\theta_{14}) \\ E(\theta_{15})
\end{bmatrix}.
$$

After parameterizing the $\boldsymbol{A}$ matrix, the endogenous variables in (7.53) can be solved for any exogenous shock $\boldsymbol{b}$ as

$$
(7.54)\quad \boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{b}.
$$

As we have done throughout this book, we follow Gardner's (1988) parameterization and assume that the own-price elasticity of demand, η^D, is –0.60. The own-price elasticities of input supply, ε_1 and ε_2, are 0.20 and 1.0, respectively, while the factor shares of x_1 (K_1) and x_2 (K_2) are 0.30 and 0.70. Assuming $\sigma_{12} = \sigma_{21} = 1.0$, then

$$
\sigma_{11} = -\frac{K_2\sigma_{12}}{K_1} = -\frac{(0.70)(1.0)}{0.30} = -2.33
$$

$$
\sigma_{22} = -\frac{K_1\sigma_{21}}{K_2} = -\frac{(0.30)(1.0)}{0.70} = -0.429.
$$

In addition, we use the average import and export shares of the U.S. beef market over the past decade so that $\mathcal{R}_E = 0.08$ and $\mathcal{R}_I = 0.12$. Therefore, $\mathcal{R}_s = 0.96$. The United States is a major beef importer and foreign countries readily supply the market with imports. In fact, the U.S. imposes tariff-rate quotas on beef imports from Argentina, Australia, New Zealand, Uruguay, and others as a means of supporting the domestic industry. Hence, we assume that the own-price elasticity of beef import supply, ε_I, is relatively elastic (2.0). Tani and Kusakari (2018) estimate that consumers in Japan, a major importer of U.S. beef, have a relatively inelastic own-price elasticity of demand for imported beef of −0.27. Therefore, we use that estimate for the own-price elasticity of demand for U.S. beef exports, η_E.

Import Tariffs

Suppose that the United States decided to impose a 10% *ad valorem* tariff on imported beef. This would be represented by setting $E(\theta_{14}) = -0.10$ in vector $\boldsymbol{b}$ of (7.53) because the tariff causes the price received by foreign producers who export beef to the United States to decline below the price received by U.S. producers. All other entries in the vector are set equal to 0. The following changes in the endogenous variables are obtained by solving (7.54) such that

$$\text{(7.55)} \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_1^S) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(q_I) \\ E(p^I) \\ E(q_E) \\ E(p^E) \end{bmatrix} = \begin{bmatrix} -0.010 \\ 0.016 \\ 0.011 \\ 0.016 \\ 0.004 \\ 0.013 \\ 0.022 \\ 0.013 \\ 0.004 \\ 0.013 \\ 0.022 \\ 0.013 \\ -0.168 \\ -0.084 \\ -0.004 \\ 0.016 \end{bmatrix}.$$

The import tariff causes an 8.4% decline in the prices that foreign producers receive for beef that they export to the United States, $E(p^I)$, and a 16.8% reduction in beef imports, $E(q_I)$. The U.S. price of beef, $E(p^D)$ and $E(p^S)$, increases by 1.6%. Note that the sum of the absolute values of these two price changes represents the 10% tariff placed on U.S. beef imports (8.4% + 1.6% = 10%). Higher U.S. beef prices reduce the domestic quantity demanded of beef, $E(q^D)$, by 1.0% and U.S. beef producers increase output, $E(q^S)$, by 1.1%. In addition, U.S. beef export prices, $E(p^E)$, increase by 1.6% because of the assumption that domestic, imported, and exported beef are of similar quality. This assumption could be relaxed by expanding the EDM. Although domestic production increases and export quantities decline, as shown by $E(q_E) = -0.4\%$, the 16.8% reduction in beef imports causes the overall amount of beef available to consumers to decline which explains the domestic price increase. The increase in domestic production increases the quantities and prices of both inputs.

The EDM results indicate that the import tariff helps domestic beef producers. The tariff reduces competition in the domestic market while increasing domestic prices and the demand for inputs used to produce beef. Foreign beef producers are harmed by the tariff as the price and quantity of beef supplied to the United States declines, and U.S. consumers are harmed by the tariff as consumers pay a higher price for beef and consume less beef. A further result is that consumers of U.S. beef in export country destinations are harmed by the import tariff as the price they pay for beef exported by the United States increases and their consumption declines.

Changes in an Existing Import Tariff

Suppose that a 30% import tariff on imported beef already exists. Using a domestic price of \$5.00/pound, a 30% tariff would result in a price of \$6.50 per pound for imported beef. Thus, the initial value of $\frac{p^D}{p^I}$ in (7.34) would equal 0.77. Consequently, the ***A*** matrix in (7.53) becomes

$$
\begin{bmatrix}
1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\varepsilon_I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\eta_E \\
1 & 0 & -\mathcal{R}_S & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{R}_I & 0 & \mathcal{R}_E & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & -0.77 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -p^D/p^E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{7.56}
$$

Suppose the U.S. government decides to increase the import tariff by another 10 percentage points. After setting $E(\theta_{14}) = -0.10$ and all other exogenous shocks to 0 in vector ***b*** in (7.54), the solution yields the following changes in the endogenous variables:

$$(7.57)\qquad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_1^S) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(q_I) \\ E(p^I) \\ E(q_E) \\ E(p^E) \end{bmatrix} = \begin{bmatrix} -0.010 \\ 0.017 \\ 0.011 \\ 0.017 \\ 0.005 \\ 0.014 \\ 0.023 \\ 0.014 \\ 0.005 \\ 0.014 \\ 0.023 \\ 0.014 \\ -0.174 \\ -0.087 \\ -0.004 \\ 0.017 \end{bmatrix}.$$

Most of the results are similar to those in the previous section; changes in all of the endogenous variables are only slightly larger because of the relatively small increase in the import tariff from 30% to 40%.

Export Subsidies

Suppose that the United States decided to impose a 10% *ad valorem* export subsidy on exported beef to support the domestic beef industry. This action would place a price wedge between export and domestic prices and cause U.S. beef export prices to be lower than U.S. domestic beef prices. Lower export prices would increase the quantity demanded of U.S. beef in U.S. export markets. Hence, (7.51) is the appropriate mechanism for including this price wedge. Note that setting $E(\theta_{15}) = -0.10$ in (7.51) causes the beef export price to be 10% lower than the domestic beef price. Therefore, $E(\theta_{15})$ is set equal to -0.10 in vector $\boldsymbol{b}$ of (7.53) and all other entries are set equal to 0. The following changes in the endogenous variables $\boldsymbol{y}$ are obtained by solving (7.54):

$$(7.58)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_1^S) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(q_I) \\ E(p^I) \\ E(q_E) \\ E(p^E) \end{bmatrix} = \begin{bmatrix} -0.001 \\ 0.001 \\ 0.001 \\ 0.001 \\ 0.001 \\ 0.001 \\ 0.002 \\ 0.001 \\ 0.001 \\ 0.001 \\ 0.002 \\ 0.001 \\ 0.003 \\ 0.001 \\ 0.027 \\ -0.099 \end{bmatrix}.$$

The results indicate that the domestic consumption of beef, $E(q^D)$, declines by 0.1% because the price of beef, $E(p^D)$ and $E(p^S)$, increases by 0.1%. The quantity of domestic beef produced, $E(q^S)$, increases by 0.1% because the export subsidy increases export quantities, $E(q_E)$, by 2.7%. The subsidy reduces the price of exports, $E(p^E)$, by 9.9%. Note that the difference between lower export prices and higher domestic prices is equal to the size of the 10% export subsidy. The price of beef imports, $E(p^I)$, increases by 0.1% in response to higher domestic prices, and the quantity of beef imports, $E(q_I)$, increases by 0.3%. The increase in domestic production increases the prices and quantities of both inputs.

Import Quantity Restrictions

The preceding examples were operationalized by shocking a single equation. This approach can be used whenever the wedge being created between prices or quantities is known with certainty. For example, a 10% tax applied to the output of an industry places a 10% wedge between consumer and producer prices. The incidence of the tax, however, is endogenous and depends upon relative supply and demand elasticities. Some policy and economic shocks produce wedges between prices or quantities that are themselves endogenous to the system. For example, if a government institutes an import restriction in the form of a quota, then the price impacts of this policy are endogenously determined. The European Union has used various methods to restrict imports of beef products from many nations, including Argentina, Australia, Brazil, Canada, New Zealand, Paraguay, Uruguay, and the United States.

For an import quantity restriction, a new equation must be added to the EDM modeled in (7.36)–(7.51). Because an import quantity restriction creates an unknown price wedge between the price of domestic beef, $E(p^D)$, and the price of imported beef, $E(p^I)$, $E(\theta_{14})$ becomes an endogenous variable such that (7.50) is now written as

$$(7.59)\quad E(p^I) - \left(\frac{p^D}{p^I}\right) E(p^D) - E(\theta_{14}) = 0.$$

An additional equation is added to the system to represent the import quantity restriction policy:

$$(7.60)\quad E(q_I) = E(\psi_1).$$

If a 10% quantity import restriction is established, then $E(\psi_1) = -0.10$. This value indicates that while foreign producers receive the domestic price for imported beef, the marginal cost of beef in foreign countries is reduced relative to the price of beef in the United States. The new system of equations now consists of (7.36)–(7.49), (7.59), (7.51), and (7.60). Using linear algebra, the system of equations is written as

$$(7.61)\quad \begin{bmatrix}
1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 1 & -K_1\sigma_{21} & -K_2\sigma_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\varepsilon_I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\eta_E & 0 \\
1 & 0 & -\mathcal{R}_S & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{R}_I & 0 & \mathcal{R}_E & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & -p^D/p^I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & -p^D/p^E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_1^S) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(q_I) \\ E(p^I) \\ E(q_E) \\ E(p^E) \\ E(\theta_{14})
\end{bmatrix}
=
\begin{bmatrix}
E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ 0 \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\theta_{13}) \\ 0 \\ E(\theta_{15}) \\ E(\psi_1)
\end{bmatrix}.$$

The model represented by (7.61) is used in place of (7.53). Thus, the solution to (7.54) generates changes in the endogenous variables $\boldsymbol{y}$ for any exogenous shock $\boldsymbol{b}$. After parameterizing the $\boldsymbol{A}$ matrix using the values noted above, the 10% quantity restriction on input 1 is introduced by setting $E(\psi_1) = -0.10$ in vector $\boldsymbol{b}$ with all other entries set equal to 0. The solution to (7.61) yields the following changes in the endogenous variables $\boldsymbol{y}$:

$$(7.62)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(w_1^D) \\ E(w_1^S) \\ E(x_1^S) \\ E(x_2^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(q_I) \\ E(p^I) \\ E(q_E) \\ E(p^E) \\ E(\theta_{14}) \end{bmatrix} = \begin{bmatrix} -0.006 \\ 0.010 \\ 0.006 \\ 0.010 \\ 0.003 \\ 0.008 \\ 0.013 \\ 0.008 \\ 0.003 \\ 0.008 \\ 0.013 \\ 0.008 \\ -0.100 \\ -0.050 \\ -0.003 \\ 0.010 \\ -0.060 \end{bmatrix}.$$

The 10% import quantity restriction increases domestic prices, $E(p^D)$, by 1.0% and reduces domestic beef consumption, $E(q^D)$, by 0.6%. The quantity of beef imports, $E(q_I)$, declines by the legislated 10% amount. The price of imported beef, $E(p^I)$, decreases by 5%, while the price of U.S. beef exports increases by 1%. The shadow value, $E(\theta_{14})$, of the price wedge is 6%, which is the sum of the decline in the price of beef imports and the increase in domestic beef prices. The domestic production of beef, $E(q^S)$, increases by 0.6% and the domestic producer price of beef, $E(p^S)$, increases by 1%. The price of beef exports, $E(p^E)$, also increases by 1% given the assumption that exported beef is of similar quality and price to domestic beef. The increase in exported beef price decreases the consumption of U.S. beef exports in other countries, $E(q_E)$, by 0.3%. Input demand quantities and prices increase.

One Output with Three Inputs

In this section, we model a market in which three inputs are used to produce a single output. The EDM is written as

(7.63) $E(q^D) = \eta^D E(p^D) + E(\theta_1)$

(7.64) $E(p^S) = K_1 E(w_1^D) + K_2 E(w_2^D) + K_3 E(w_3^D) + E(\theta_2)$

(7.65) $E(x_1^D) = E(q^S) + K_1\sigma_{11}E(w_1^D) + K_2\sigma_{12}E(w_2^D) + K_3\sigma_{13}E(w_3^D) + E(\theta_3)$

(7.66) $E(x_2^D) = E(q^S) + K_1\sigma_{21}E(w_1^D) + K_2\sigma_{22}E(w_2^D) + K_3\sigma_{23}E(w_3^D) + E(\theta_4)$

(7.67) $E(x_3^D) = E(q^S) + K_1\sigma_{31}E(w_1^D) + K_2\sigma_{32}E(w_2^D) + K_3\sigma_{33}E(w_3^D) + E(\theta_5)$

(7.68) $E(x_1^S) = \varepsilon_1 E(w_1^S) + E(\theta_6)$

(7.69) $E(x_2^S) = \varepsilon_2 E(w_2^S) + E(\theta_7)$

(7.70) $E(x_3^S) = \varepsilon_3 E(w_3^S) + E(\theta_8)$.

Allowing for potential price and quantity wedges between demand and supply output quantities and prices, and between input demand and supply quantities and prices, the following equilibrium conditions are assumed:

(7.71) $E(q^D) = E(q^S) + E(\theta_9)$

(7.72) $E(x_1^D) = E(x_1^S) + E(\theta_{10})$

(7.73) $E(x_2^D) = E(x_2^S) + E(\theta_{11})$

(7.74) $E(x_3^D) = E(x_3^S) + E(\theta_{12})$

(7.75) $E(p^D) = E(p^S) + E(\theta_{13})$

(7.76) $E(w_1^D) = E(w_1^S) + E(\theta_{14})$

(7.77) $E(w_2^D) = E(w_2^S) + E(\theta_{15})$

(7.78) $E(w_3^D) = E(w_3^S) + E(\theta_{16})$

Moving the endogenous variables of (7.63)–(7.78) to the left-hand side yields

(7.79) $E(q^D) - \eta^D E(p^D) = E(\theta_1)$

(7.80) $E(p^S) - K_1 E(w_1^D) - K_2 E(w_2^D) - K_3 E(w_3^D) = E(\theta_2)$

(7.81) $E(x_1^D) - E(Q^S) - K_1\sigma_{11} E(w_1^D) - K_2\sigma_{12} E(w_2^D) - K_3\sigma_{13} E(w_3^D) = E(\theta_3)$

(7.82) $E(x_2^D) - E(Q^S) - K_1\sigma_{21} E(w_1^D) - K_2\sigma_{22} E(w_2^D) - K_3\sigma_{23} E(w_3^D) = E(\theta_4)$

(7.83) $E(x_3^D) - E(Q) - K_1\sigma_{31} E(w_1^D) - K_2\sigma_{32} E(w_2^D) - K_3\sigma_{33} E(w_3^D) = E(\theta_5)$

(7.84) $E(x_1^S) - \varepsilon_1 E(w_1^S) = E(\theta_6)$

(7.85) $E(x_2^S) - \varepsilon_2 E(w_2^S) = E(\theta_7)$

(7.86) $E(x_3^S) - \varepsilon_3 E(w_3^S) = E(\theta_8)$

(7.87) $E(q^D) - E(q^S) = E(\theta_9)$

(7.88) $E(x_1^D) - E(x_1^S) = E(\theta_{10})$

(7.89) $E(x_2^D) - E(x_2^S) = E(\theta_{11})$

(7.90) $E(x_3^D) - E(x_3^S) = E(\theta_{12})$

(7.91) $E(p^D) - E(p^S) = E(\theta_{13})$

(7.92) $E(w_1^D) - E(w_1^S) = E(\theta_{14})$

(7.93) $E(w_2^D) - E(w_2^S) = E(\theta_{15})$

(7.94) $E(w_3^D) - E(w_3^S) = E(\theta_{16})$.

The exogenous shocks represented by θ_1–θ_{16} can be used to evaluate many policy interventions or other exogenous shocks to the system. The choice of shock depends upon the specific application. However, shocks to production technologies θ_2, θ_3, θ_4, or θ_5 are not independent of each other. Putting (7.79)–(7.94) into matrix notation yields

(7.95) $\boldsymbol{Ay} = \boldsymbol{b}$,

where $\boldsymbol{A}$ is a 16×16 matrix of parameters, $\boldsymbol{y}$ is a 16×1 vector of endogenous variables, and $\boldsymbol{b}$ is a 16×1 vector of exogenous shocks such that

$$(7.96)\quad \begin{bmatrix} 1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -K_1 & -K_2 & -K_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & -K_3\sigma_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & -K_1\sigma_{21} & -K_2\sigma_{22} & -K_3\sigma_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & -K_1\sigma_{31} & -K_2\sigma_{32} & -K_3\sigma_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_3 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \end{bmatrix} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\theta_{13}) \\ E(\theta_{14}) \\ E(\theta_{15}) \\ E(\theta_{16}) \end{bmatrix}.$$

After parameterizing the $\boldsymbol{A}$ matrix, the endogenous variables in (7.96) can be solved for any exogenous shock $\boldsymbol{b}$ as

$$(7.97)\quad \boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{b}.$$

For the following examples, the $\boldsymbol{A}$ matrix is parametrized using Gardner's (1988, p. 114) example with the own-price elasticity of demand, η^D, equal to −0.40. The own-price elasticities of input supply (ε_1, ε_2, ε_3) are 2.0, 1.0, and 0.2, respectively, while the factor shares of x_1 (K_1), x_2 (K_2), and x_3 (K_3) are 0.20, 0.50, and 0.30 and sum to 1. Assuming $\sigma_{12} = \sigma_{21} = 0.30$, $\sigma_{13} = \sigma_{31} = 1.0$, and $\sigma_{23} = \sigma_{32} = 0.50$, the remaining AES values are

$$\sigma_{11} = -\frac{(K_2\sigma_{12})+(K_3\sigma_{13})}{K_1} = -2.25$$

$$\sigma_{22} = -\frac{(K_1\sigma_{21})+(K_3\sigma_{23})}{K_2} = -0.42$$

$$\sigma_{33} = -\frac{(K_1\sigma_{31})+(K_2\sigma_{32})}{K_3} = -1.50.$$

Homogeneity of Degree 0

A useful exercise to test for the consistency of an EDM is to consider the homogeneity of the production technology. For example, if the production technology in (7.79)–(7.94) is homogeneous of degree 0 (HD0) in input and output prices, then equal percentage changes in input prices coupled with an identical percentage change in output price should have no impact on equilibrium quantities.

The test involves increasing input prices by, for example 10%, with a concurrent and equal increase in output prices. Essentially, this is equivalent to taxing the price of inputs and subsidizing the price of output. To implement the test, the values $E(\theta_{14}) = 0.10$, $E(\theta_{15}) = 0.10$, and $E(\theta_{16}) = 0.10$ are entered into vector $\boldsymbol{b}$ in (7.96). The input price wedges are entered as positive numbers because the "tax" causes input supply prices to be lower than input demand prices. The increase in output price is modeled as an output price subsidy to producers such that $E(\theta_{13}) = -0.10$ is entered in vector $\boldsymbol{b}$ of (7.96). That is, the output price subsidy would cause the supply price to be 10% larger than the output demand price. After setting all other entries in the vector to 0, the following changes in the endogenous variables $\boldsymbol{y}$ are obtained by solving (7.96) using (7.97):

$$\text{(7.98)} \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.10 \\ 0 \\ 0 \\ 0 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The results show that a 10% increase in input demand prices and a concurrent 10% increase in the output supply price have not changed any other endogenous variable. Note that no change in output has occurred, which is the expected result for HD0 production technologies. All three input supply prices are 10% higher than input supply prices. But, as expected for HD0 production technologies, no change in the quantities of inputs used occurs.

An Exogenous Change in Demand

Consider a case in which demand for an output increases by 10%. In the case of beef, this increase could be the result of an increase in per capita incomes. The exogenous shock would be entered in the $\boldsymbol{b}$ vector of (7.96) as $E(\theta_1) = 0.10$, and all other entries would be set equal to 0. The following changes in the endogenous variables are obtained by solving (7.96):

$$(7.99)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \end{bmatrix} = \begin{bmatrix} 0.064 \\ 0.089 \\ 0.064 \\ 0.089 \\ 0.096 \\ 0.073 \\ 0.028 \\ 0.048 \\ 0.073 \\ 0.142 \\ 0.096 \\ 0.073 \\ 0.028 \\ 0.048 \\ 0.073 \\ 0.142 \end{bmatrix}.$$

The results indicate that output demand quantity, $E(q^D)$, and supply quantity, $E(q^S)$, increase by 6.4%, while the equilibrium output demand, $E(p^D)$, and supply price, $E(p^S)$, increase by 8.9%. The use of input 1 increases by 9.6%, while its price increases by 4.8%. The use of input 2 increases by 7.3%, while its price increases by the same percentage because of the assumed unitary elasticity of demand for input 2. The use of input 3 increases by 2.8%, while its price increases by 14.2%.

An Increase in Demand and Fixed Input Proportions

Suppose that fixed input proportions technologies exist between inputs 1 and 3, such that $\sigma_{13} = \sigma_{31} = 0$. Assuming that $\sigma_{12} = \sigma_{21} = 0.30$ and $\sigma_{23} = \sigma_{32} = 0.50$, the remaining AES values are calculated as

$$\sigma_{11} = -\frac{(K_2\sigma_{12})+(K_3\sigma_{13})}{K_1} = -0.75$$

$$\sigma_{22} = -\frac{(K_1\sigma_{21})+(K_3\sigma_{23})}{K_2} = -0.42$$

$$\sigma_{33} = -\frac{(K_1\sigma_{31})+(K_2\sigma_{32})}{K_3} = -0.83.$$

Once again, assume that an exogenous 10% increase in the demand for output occurs. The exogenous shock is entered in vector $\boldsymbol{b}$ of (7.96) as $E(\theta_1) = 0.10$, and all other values in the vector are set equal to 0. The changes in the endogenous variables are obtained by solving (7.95) which results in

$$\text{(7.100)} \quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \end{bmatrix} = \begin{bmatrix} 0.061 \\ 0.097 \\ 0.061 \\ 0.097 \\ 0.067 \\ 0.074 \\ 0.035 \\ 0.034 \\ 0.074 \\ 0.177 \\ 0.067 \\ 0.074 \\ 0.035 \\ 0.034 \\ 0.074 \\ 0.177 \end{bmatrix}.$$

The results indicate that the output demand quantity, $E(q^D)$, and supply quantity, $E(q^S)$, increase by 6.1%, while the equilibrium output demand price, $E(p^D)$, and supply price, $E(p^S)$, increase by 9.7%. Because of the assumption of fixed input proportions between inputs 1 and 3, the increase in output is smaller and the increase in output price is larger compared to a 10% increase in demand when variable input proportions exist. The use of input 1 increases by 6.7%, while its price increases by 3.4%. The use of input 2 increases by 7.4%, while its price increases by the same percentage because of the assumed unitary own-price elasticity of supply for the input. The use of input 3 increases by 3.5%, while its price increases by 17.7%.

Endogenous Wedges Caused by Policy Changes

As noted previously, many exogenous shocks are operationalized in EDMs by shocking one or more behavioral equations. However, it is often the case that a policy imposes unknown wedges between prices or quantities in a market. In these cases, an additional restriction that reflects these endogenous effects must be added to an EDM.

Restriction on Output

Consider the one-output, three-input model with variable input proportions presented above and assume that a policy is instituted that restricts output by 10%. This policy would create an unknown wedge between the prices paid by consumers, $E(p^D)$, and the prices received by producers, $E(p^S)$. Although producers receive the higher consumer price for their production, the reduction in quantity also means that societal costs are below the willingness to pay for the reduced output. These costs are represented by $E(p^S)$. Therefore, (7.91) is changed so that the exogenous shock, $E(\theta_{13})$, becomes an endogenous variable,

$$E(p^D) - E(p^S) - E(\theta_{13}) = 0, \tag{7.101}$$

and an additional equation is added that represents the policy-induced reduction in output, such that

$$E(q^S) = E(\psi_1). \tag{7.102}$$

The new system of equations now consists of (7.79)–(7.90), (7.101), (7.92)–(7.94), and (7.102) such that (7.96) becomes

$$\begin{bmatrix}
1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -K_1 & -K_2 & -K_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & -K_3\sigma_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 & 0 & -K_1\sigma_{21} & -K_2\sigma_{22} & -K_3\sigma_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 1 & -K_1\sigma_{31} & -K_2\sigma_{32} & -K_3\sigma_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_3 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \\ E(\theta_{13})
\end{bmatrix}
=
\begin{bmatrix}
E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \\ 0 \\ E(\theta_{14}) \\ E(\theta_{15}) \\ E(\theta_{16}) \\ E(\psi_1)
\end{bmatrix}. \tag{7.103}$$

Changes in the endogenous variables are obtained by entering the restriction on output in vector $\boldsymbol{b}$ as $E(\psi_1) = -0.10$ in (7.103), setting all other values in the vector equal to 0, and parameterizing the $\boldsymbol{A}$ matrix as noted above. The changes in the endogenous variables are obtained by solving (7.103), which results in

$$(7.104)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \\ E(\theta_{13}) \end{bmatrix} = \begin{bmatrix} -0.100 \\ 0.250 \\ -0.100 \\ -0.138 \\ -0.150 \\ -0.114 \\ -0.044 \\ -0.075 \\ -0.114 \\ -0.221 \\ -0.150 \\ -0.114 \\ -0.044 \\ -0.075 \\ -0.114 \\ -0.221 \\ 0.388 \end{bmatrix}$$

The 10% legislated reduction in output causes the quantity produced, $E(q^S)$, and consumed, $E(q^D)$, to decline by 10%. The endogenously determined wedge between consumer prices and producer prices is 38.8%, as indicated by the value of $E(\theta_{13})$. The price that consumers pay, $E(p^D)$, and the price that producers actually receive for this restricted output level increase by 25%, while the marginal cost of the output, $E(p^S)$, declines by 13.8%. The sum of the absolute values of these two changes is equal to the shadow value, $E(\theta_{13})$, of 38.8%. The restricted level of output reduces the use of input 1 by 15% and its price by 7.5%, the use of input 2 decreases by 11.4% and its price declines by the same percentage. The use of input 3 declines by 4.4% and its price declines by 22.1%.

Restriction on the Use of Input 1

Consider the one-output, three-input model, and assume that a policy is instituted that restricts the use of input 1 by 10%. This policy creates an unknown wedge between $E(w_1^D)$ and $E(w_1^S)$. To accommodate this endogeneity, (7.92) is written as

$$(7.105)\quad E(w_1^D) - E(w_1^S) - E(\theta_{14}) = 0.$$

An additional equation must be added to the system to account for the additional endogenous variable, $E(\theta_{14})$, and to reflect the restriction on the use of input 1:

$$E(x_1^D) = E(\psi_1). \tag{7.106}$$

The new EDM system of equations now consists of (7.79)–(7.91), (7.105), (7.93), (7.94), and (7.106) such that

$$\begin{bmatrix}
1 & -\eta^D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -K_1 & -K_2 & -K_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1 & 0 & 1 & 0 & 0 & -K_1\sigma_{11} & -K_2\sigma_{12} & -K_3\sigma_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & 0 & 0 & 1 & 0 & -K_1\sigma_{21} & -K_2\sigma_{22} & -K_3\sigma_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & 0 & 0 & 0 & 1 & -K_1\sigma_{31} & -K_2\sigma_{32} & -K_3\sigma_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_2 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\varepsilon_3 & 0\\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
E(q^D)\\ E(p^D)\\ E(q^S)\\ E(p^S)\\ E(x_1^D)\\ E(x_2^D)\\ E(x_3^D)\\ E(w_1^D)\\ E(w_2^D)\\ E(w_3^D)\\ E(x_1^S)\\ E(x_2^S)\\ E(x_3^S)\\ E(w_1^S)\\ E(w_2^S)\\ E(w_3^S)\\ E(\theta_{14})
\end{bmatrix}
=
\begin{bmatrix}
E(\theta_1)\\ E(\theta_2)\\ E(\theta_3)\\ E(\theta_4)\\ E(\theta_5)\\ E(\theta_6)\\ E(\theta_7)\\ E(\theta_8)\\ E(\theta_9)\\ E(\theta_{10})\\ E(\theta_{11})\\ E(\theta_{12})\\ E(\theta_{13})\\ 0\\ E(\theta_{15})\\ E(\theta_{16})\\ E(\psi_1)
\end{bmatrix}. \tag{7.107}$$

Changes in the endogenous variables are obtained by entering the restriction on the use of input 1 in vector ***b*** as $E(\psi_1) = -0.10$ in (7.107), parameterizing the ***A*** matrix as noted above, and setting all other values in the vector equal to 0. Changes in the endogenous variables are obtained by solving (7.107), which results in

$$\mathbf{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \\ E(\theta_{14}) \end{bmatrix} = \begin{bmatrix} -0.019 \\ 0.048 \\ -0.019 \\ 0.048 \\ -0.100 \\ -0.002 \\ 0.006 \\ 0.199 \\ -0.002 \\ 0.031 \\ -0.100 \\ -0.002 \\ 0.006 \\ -0.050 \\ -0.002 \\ 0.031 \\ 0.249 \end{bmatrix}. \tag{7.108}$$

The 10% legislated reduction in the use of input 1 is illustrated by the 10% reduction in $E(x_1^D)$ and $E(x_1^S)$. The quantity of output declines by 1.9% and output price increases by 4.8%. The derived demand price of input 1 increases by 19.9%, while its supply price declines by 5%. The sum of the absolute values of these two price changes (19.9% + 5%) is equal to the shadow value (24.9%) as indicated by $E(\theta_{14})$. The use of input 2 declines by 0.2% and its price decreases by the same percentage. Although the use of inputs 1 and 2 decline, the use of input 3 increases by 0.6% and its price increases by 3.1%.

Financial Capital as an Input

Assume that input 3 represents financial capital or ownership equity in a production system. Further, assume that there is no substitutability between financial capital and input 1 or input 2. If an exogenous policy increases production costs, some of the impact will be absorbed by input providers in the form of lower input prices because of decreased demand for inputs. In addition, some of the additional costs are incurred by consumers in the form of higher output prices. In general, most models assume that these two sectors absorb all the costs of such policies. However, it is also the case that owners of production assets may incur some of these costs because they are residual claimants of both profits and losses in a competitive market environment. As such, financial capital is a variable input that is influenced by the production process. This balance sheet financial element changes as business activity either increases or reduces its value. Of course, financial capital can also be added to a business activity at any time.

An example of the impacts on business owners can be modeled by using the following parameterization: assume that the own-price elasticity of demand, η^D, is −0.40 while the own-price elasticities of input supply, ε_1 and ε_2, are 2.0 and 1.0, respectively. Goolsbee (1998) estimates short-run own-price elasticities of financial capital supply ranging from 1.14 to 1.74. We use the midpoint of this range (1.44) for the own-price elasticity of supply, ε_3, for equity capital.

The factor shares of x_1 (K_1), x_2 (K_2), and x_3 (K_3) are set equal to 0.20, 0.50, and 0.30, respectively, and we assume $\sigma_{12} = \sigma_{21} = 0.30$ and $\sigma_{13} = \sigma_{31} = \sigma_{23} = \sigma_{32} = 0$. This results in the following calculations for the remaining AES values:

$$\sigma_{11} = -\frac{(K_2\sigma_{12})+(K_3\sigma_{13})}{K_1} = -0.75$$

$$\sigma_{22} = -\frac{(K_1\sigma_{21})+(K_3\sigma_{23})}{K_2} = -0.12$$

$$\sigma_{33} = -\frac{(K_1\sigma_{31})+(K_2\sigma_{32})}{K_3} = 0.$$

These values are used to parameterize the one-output, three-input model described in (7.96).

An Exogenous Shock to the Costs of Input 1

Consider a case where a government policy has increased the costs of using input 1 by 10%. If input 1 was labor, such a policy could be a requirement that additional resources be spent on providing increased health or worker compensation insurance premiums. This policy would increase the price of labor at every quantity value, which would be indicated by a vertical upward shift in the inverse supply function for input 1. However, the supply function for input 1 in (7.70) is written as an ordinary supply function in which the quantity supplied of the input is the dependent variable while the independent variable is the price of input 1. Therefore, some algebraic manipulation is required to implement a 10% increase in the cost of input 1.

We begin by writing the supply function for input 1 in its general inverse form as

$$w_1^S = w_1^S(x_1^S) + \delta. \tag{7.109}$$

Totally differentiating (7.109) results in

$$dw_1^S = \frac{\partial w_1^S}{\partial x_1^S} dx_1^S + d\delta. \tag{7.110}$$

Multiplying both sides of (7.110) by $\frac{1}{w_1^S}$ and the first term on the right-hand side by $\frac{x_1^S}{x_1^S}$ results in

$$\frac{dw_1^S}{w_1^S} = \frac{\partial w_1^S}{\partial x_1^S} \frac{x_1^S}{x_1^S} \frac{dx_1^S}{w_1^S} + \frac{d\delta}{w_1^S}. \tag{7.111}$$

Rearranging terms yields

$$\frac{dw_1^S}{w_1^S} = \frac{\partial w_1^S}{\partial x_1^S} \frac{x_1^S}{w_1^S} \frac{dx_1^S}{x_1^S} + \frac{d\delta}{w_1^S}, \tag{7.112}$$

or, in proportional elasticity form,

$$E(w_1^S) = \frac{1}{\varepsilon_1} E(x_1^S) + E(\delta), \tag{7.113}$$

where ε_1 is the ordinary own-price elasticity of supply of input 1. A 10% increase in the price of input 1 for all quantity levels would be represented by setting $E(\delta)$ = 0.10 in (7.113). To implement this in the EDM, we solve (7.113) for $E(x_1^S)$ as

$$E(x_1^S) = \varepsilon_1 E(w_1^S) - \varepsilon_1 E(\delta). \tag{7.114}$$

In the previous examples, we assumed that ε_1 = 2.0. If $E(\delta)$ = 0.10, the last term in (7.114) is equal to –0.20. The policy of adding a 10% cost to the price of labor is represented by setting $E(\theta_6)$ = –0.20 in vector $\boldsymbol{b}$ of (7.96), with all other values equal to 0. Solving (7.96) for the endogenous variables $\boldsymbol{y}$ results in the following:

$$(7.115)\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \end{bmatrix} = \begin{bmatrix} -0.007 \\ 0.016 \\ -0.007 \\ 0.016 \\ -0.020 \\ -0.001 \\ -0.007 \\ 0.090 \\ -0.001 \\ -0.005 \\ -0.020 \\ -0.001 \\ -0.007 \\ 0.090 \\ -0.001 \\ -0.005 \end{bmatrix}.$$

The results indicate that the output demand quantity, $E(q^D)$, and supply quantity, $E(q^S)$, decrease by 0.70%, while the equilibrium output prices, $E(p^D)$ and $E(p^S)$, increase by 1.6%. The use of input 1 declines by 2.0%, the use of input 2 decreases by 0.10%, and the use of input 3 (i.e., financial capital) declines by 0.70%. The price of input 1 increases by 9.0%, and the price of input 2 declines by 0.10%. Note that the price of input 3, or the value of financial ownership capital, declines by 0.5%.

Summary

This chapter has shown that EDMs are sufficiently flexible to allow for a wide variety of modeling efforts. Trade policies can be incorporated into EDMs so that the impacts of tariffs, quotas, and export enhancement programs can be evaluated on domestic and international producers and consumers. Multiple output markets consisting of vertical or horizontal relationships can be included in EDMs. In addition, EDMs can be used to include multiple input markets. It is often the case that government policies are implemented in input markets to achieve a desired result in output markets. We have presented a model in which financial capital or owners' equity is included as an input. In competitive markets, owners' equity is always affected by government policies and should be included wherever possible when creating EDMs.

From: Student@UEconomics.edu
To: Professor Watson
Date: Wednesday, 30 Oct 2021 at 2:42 p.m.
Subject: Complexity of EDMs

Dear Professor,

In Chapter 7, you introduce more complex EDM models. I was thinking back to our initial discussion the first day of class about CGE vs EDM models. As EDMs get more complex, at one point are we just better off using a CGE model?

Shirley

From: Professor Watson
To: Student@UEconomics.edu
Date: Thursday, 31 Oct 2021 at 7:15 a.m.
Re: Complexity of EDMs

Dear Class,

You are right in that the more complex we make an EDM model, the more one might think about just using a CGE model. CGE models, however, can be more complex than you may realize, as they often consist of hundreds of equations. Modifying or building such models is very expensive, and error checking the programming is not only tedious but often impossible. EDMs can evaluate the effects of specific polices on vertical and/or horizontal sectors and in multiple input and output markets. The limitation or constraint on EDMs is often linked with data availability, including estimates of input supply elasticities, Allen elasticities of substitution, and factor shares. One of the reasons EDMs have been widely used in agriculture and resource research is because data are often available from sources such as the USDA. In addition, many trade policy issues occur within the agricultural sector. I am giving you the tools to develop more complex EDM models, which will help inform you as to the types of data that you need. Obviously, one should use simpler models when possible. As you build EDMs, it is important to think about the policy you are interested in modeling. As we have done in class, it is often valuable to use supply and demand diagrams to fully understand policy implications. This allows for the appropriate modeling of such shocks. Then, you can consider data availability as you build an EDM.

All the best,
Dr. Watson

»Chapter Eight

CONSUMER SURPLUS, PRODUCER SURPLUS, AND DEADWEIGHT LOSSES

Equilibrium displacement models (EDMs) have a limited ability to estimate changes in consumer surplus, producer surplus, and deadweight losses. However, this is true for all partial equilibrium economic models that use arbitrary demand and supply functions, rather than those obtained from underlying utility functions, to estimate these changes. In addition, such estimates can be obtained from EDMs only within the confines of the sectors for which endogenous responses are specifically included in the model. Although EDMs are used to estimate changes in price and quantity equilibria caused by exogenous economic or policy shocks emanating from outside of the sectors being modeled, one must be cautious when using EDMs to evaluate changes in consumer or producer surplus induced by shocks that are exogenous to the model's behavioral equations.

Estimates of changes in producer and consumer surplus and, when appropriate, deadweight losses generally have limited value as stand-alone metrics. That is, these measures are usually most meaningful when compared to a base or standard. Unless one uses functional forms derived from a specific utility function, initial levels of total consumer and producer surplus are unknown, which makes values (e.g., a percentage reduction in consumer surplus) nonestimable. Consequently, we consider changes in surpluses or deadweight losses relative to the initial size of an industry. We have chosen to use total revenue or total costs of a sector at initial equilibrium prices and quantities as a base. Other standards could be used as well.

Surplus Measures and Partial Equilibrium Analyses

In partial equilibrium analyses, researchers must identify exogenous versus endogenous components of the economic system being modeled. As shown in previous chapters, EDMs specify endogenous elements of a system in a general form and then approximate the effects of exogenous shocks to that system. In general, partial equilibrium models and EDMs are only able to evaluate the surplus effects of shocks that generate price-quantity movements along the model's internally determined equilibrium trajectories or total-response functions. As we discuss below, an EDM cannot be used to estimate surplus effects for a "partially external" sector whose given total-response function changes due to factors that are external to the EDM.

For example, Figure 8.1 presents a schematic for an EDM that includes a single endogenous supply and demand sector and three exogenous components: demand shifters, supply shifters, and tax policies. The terms δ^D and δ^S represent exogenous shocks to the demand and supply functions, respectively, while τ represents an *ad valorem* tax. In equilibrium, the demand and supply prices and quantities are represented by p_0 and q_0. Hence, own-price and quantities are endogenous to the system.

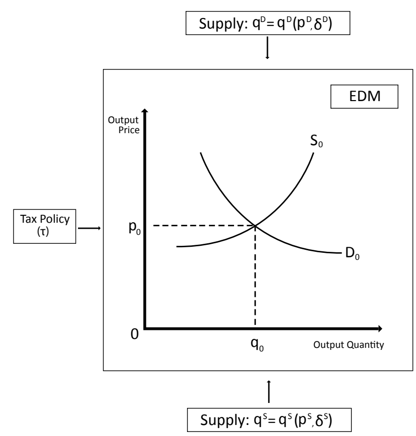

Figure 8.1. Exogenous and Endogenous Elements of an EDM

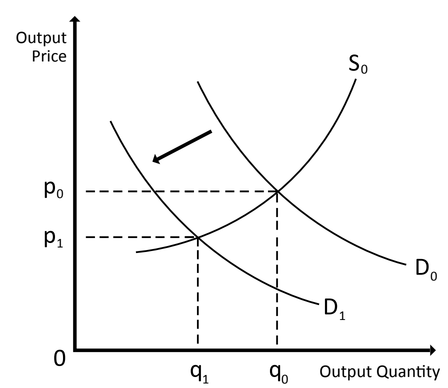

Figure 8.2. An Exogenous Decrease in Demand

Assume an exogenous shock in demand, δ^D, shifts D_0 downward to D_1 as noted by the arrow in Figure 8.2. An EDM can be used to estimate p_1 and q_1 and changes in producer surplus because these values are endogenous to the system. However, the model cannot be used to estimate changes in consumer surplus caused by an exogenously induced demand shift. That is, without endogenizing supply and demand information for consumer substitute goods, changes in income, or other potential sources of the exogenous demand shock, we are unable to determine whether total consumer surplus has increased or decreased. Although the area under the demand curve declines for the consumer good illustrated in Figure 8.2, the lower price for the good changes the areas under the demand curves for other consumer goods not included in the model. For example, the decrease in demand indicated in Figure 8.2 could have been caused by an increase in the demand for a substitute good. Hence, it is unknown whether the reduction in consumer surplus associated with a decrease in demand presented in Figure 8.2 is larger or smaller than changes in consumer surplus that result from interactions among substitutes or complementary goods.

Total-Response Functions, Surplus Measures, and Equilibrium Trajectories

As discussed in previous chapters, an EDM's endogenous supply and demand relations are total-response functions that represent net quantity-price responses after accounting for all interactions and feedback effects among an EDM's internal equations. For example, if a supply shock changes the price for the good in Figure 8.1, the demand relationship, D_0, is assumed to represent the net change in the quantity demanded of that good after accounting for all interactions in the consumer's exogenous economic environment including changes in the prices of other goods. The assumed inclusion of feedback effects in the system's total-response supply relations and total-response demand functions invokes the concepts of total elasticities (Tomek and Robinson, 1990; Cochrane, 1955; Buse, 1958). An EDM's total-response supply and demand functions allow for the valid estimation of changes in surplus provided that the changes are caused by factors included in the system. Surplus changes are represented as areas under or above total-response functions when changes in quantities and prices are induced by shocks endogenous to the EDM system (Just, Hueth, and Schmitz, 2004).

Surplus Effects of a Tax or Subsidy in the Output Market

Equations (5.2)–(5.8) presented a one-output, two-input EDM that allowed for estimating the impact of a 10% excise tax placed on the output market. The total-response supply and demand functions intersect at the initial pretax equilibrium point (p_0, q_0) as illustrated in Figure 8.3. The EDM shows that a tax on the output market causes the prices that consumers pay to increase to p_t^D and the price that producers receive for their production to decline to p_t^S. The difference between these two values represents a tax wedge between the two prices. The new equilibrium quantity declines to q_t. Note that the new price and quantity equilibrium occurs along the solid arrows, which represent the EDM's estimated equilibrium trajectories, rather than along the actual supply and demand curves.

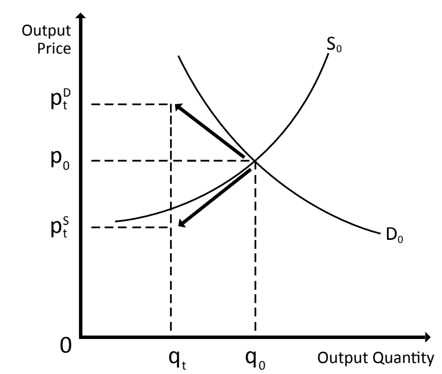

Figure 8.3. An Excise Tax in the Output Market

The imposition of the output tax does not shift either the demand or supply functions. The arrows represent the EDM linearly approximated equilibria trajectories caused by the tax from the initial equilibrium point to the new price and quantity levels. The EDM traces the price and quantity trajectories caused by the tax while accounting for feedback effects among the endogenous prices and quantities included in the model's behavioral equations.

It is likely that most, if not all, total-response demand and supply functions are nonlinear. An EDM represents a first-order linear approximation of the true underlying price and quantity trajectories using assumed or estimated total-response elasticities evaluated at the initial equilibrium point (Figure 8.3). We cannot estimate changes in surplus by integrating the true total-response functions without knowing their functional forms. However, we can obtain approximate changes in surplus by computing areas under or above the linear EDM equilibria trajectories. Note that these areas geometrically consist of triangles or rectangles. Nonetheless, we emphasize that the estimated areas are only as accurate as the degree to which the linear approximations represent the true underlying, but unknown, nonlinear functions. As a result, EDM surplus estimates are more accurate when calculated for small exogenous shocks.

With this caveat, EDMs can be used to estimate changes in consumer and producer surplus that result from changes in equilibria. If appropriate, tax receipts and deadweight losses or the incidence of both can be estimated from EDM results. Because we cannot integrate the unknown total-response functions to obtain initial total consumer or producer surplus, we need a standardizing metric to make comparisons. It seems reasonable to consider these changes in terms of the size of an industry. We initially use the total revenue of the industry being modeled at the initial equilibrium price and quantity levels as the standardizing metric. Of course, this is not the only base with which to make relative comparisons. However, comparing surplus changes to the initial size of the industry is a convenient way of measuring percentage changes in surplus that result from exogenous economic or policy shocks.

New equilibria caused by a change in the demand or supply of a product can be obtained from an EDM. However, changes in surplus are not estimable for exogenous shocks that shift demand or supply functions if those shocks emanate from outside the modeling structure. Just, Hueth, and Schmitz (2004) note that the concurrent aggregate or total change in producer surplus of input suppliers can be obtained directly from changes in the output market. Alternatively, changes in producer surplus for each of the input markets can be estimated and summed. If the underlying functional forms of the system of supply and demand equations are known with certainty, then the two approaches provide equivalent results. We show that the two approaches to estimating changes in producer surplus are not likely to be exact because of the EDM's linear approximations. However, they are reasonably close to each other in most circumstances.

We present procedures for estimating surplus calculations for a single output that is produced using two inputs. The results are generalizable to n inputs for calculating changes in consumer surplus, producer surplus, and—where appropriate—deadweight losses and tax receipts. We use total revenue, which is the product of the initial price, p_0, and quantity, q_0, as a measure of industry size. Therefore, our estimates of changes in producer and consumer surplus are proportional to this value. However, we also provide procedures that allow for scaling based on the size of each input sector. We note that homogeneity of degree 1 (HD1) in input and output production functions are essential for the development of a theoretically consistent EDM. In addition, the assumption of perfect competition in which producers are price takers implies zero economic profits. Thus, total revenue of the industry (p_0q_0) equals the total costs of the inputs used by the industry, such that (p_0q_0) = $(\boldsymbol{w}_0\boldsymbol{x}_0) = (w_{1,0}x_{1,0} + w_{2,0}x_{2,0} + \cdots + w_{n,0}x_{n,0})$.

Changes in Consumer and Producer Surplus within the Output Market

Figure 8.4 illustrates an EDM's linearly approximated equilibria trajectories resulting from the imposition of an excise tax on the output market, which was more broadly illustrated in Figure 8.3. The nonlinear supply and demand depictions intersect at the initial equilibrium point. The EDM of (5.2)–(5.8) provides new equilibrium consumer and producer prices and output levels. For illustrative clarity, Figure 8.4 is developed without the initial demand and supply curves presented in Figure 8.3. Figure 8.4 shows that these new EDM equilibrium estimates result from movements along the linear equilibrium trajectories, indicated by the arrows. The price paid by consumers increases to p_t^D, the price received by producers declines to p_t^S, and the quantity supplied and demanded declines to q_t. The absolute values of the size of these changes are given by $|dp^D|$, $|dp^S|$, and $|dq|$, respectively. The absolute value of the change in consumer surplus can be approximated as area $(a + b)$ in Figure 8.4, or

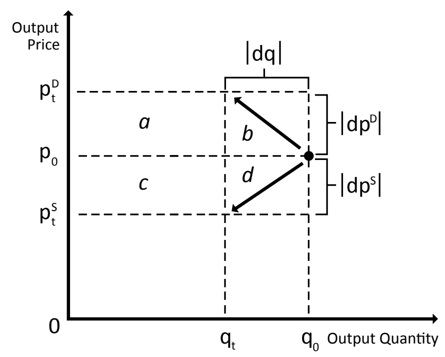

Figure 8.4. Equilibria Trajectories Resulting from an Excise Tax

(8.1) $$|\Delta CS| = (|dp^D| * q_0) - (½|dp^D||dq|),$$

$$= \left(\frac{|dp^D|}{p_0} * p_0q_0\right) - \left(½\frac{|dp^D|}{p_0}\frac{|dq|}{q_0}\right)p_0q_0,$$

$$= [(|E(p^D)|)\{1 - ½|E(q)|\}]\,p_0q_0.$$

In this example, the change in consumer surplus is negative because $E(p_D) > 0$ and $E(q) < 0$, given that $E(q)$ can never be larger (in absolute value) than –1 or –100%. Of course, an EDM is probably an inappropriate approach for estimating changes that would result in quantities being altered by such a large amount. Hence, ΔCS = –area $(a + b)$, such that

$$(8.2) \quad \Delta CS = -\left[\left(E(p^D)\right)\{1 - ½(-1)E(q)\}\right]p_0 q_0$$

or

$$(8.3) \quad \Delta CS = -\left[\left(E(p^D)\right)\{1 + ½E(q)\}\right]p_0 q_0.$$

Equation (8.3) measures the reduction in consumer surplus caused by the tax measured as a percentage of the total revenue of the output market valued at the initial equilibrium point. Note that the change in consumer surplus would be positive if the effects of an output subsidy are being evaluated because, in that case, $E(p_D) < 0$ and $E(q) > 0$.

The absolute value of the change in producer surplus caused by a tax on the output market is approximated by area $(c + d)$ in Figure 8.4:

$$(8.4) \quad \left|\Delta PS_Q\right| = (|dp^S| * q_0) - (½|dp^S||dq|)$$

or

$$(8.5) \quad \left|\Delta PS_Q\right| = [(|E(p^S)|)\{1 - ½|E(q)|\}]\, p_0 q_0.$$

with the subscript Q indicating that producer surplus is being calculated in the output market. The tax on output causes $E(p_S) < 0$ and $E(q) < 0$ so that the change in producer surplus is negative. Therefore, ΔPS_Q = –area $(c + d)$ is

$$(8.6) \quad \Delta PS_Q = -\left[-\left(E(p^S)\right)\{1 - ½(-1)E(q)\}\right]p_0 q_0$$

or

$$(8.7) \quad \Delta PS_Q = \left[\left(E(p^S)\right)\{1 + ½E(q)\}\right]p_0 q_0,$$

which measures the reduction in producer surplus as a proportion of the industry's initial total revenue. Note that for a tax, (8.7) is negative as $E(p^S) < 0$, and the largest negative value that $E(q)$ could be is 100%. The change in producer surplus is positive for a subsidy because both $E(p^S)$ and $E(q)$ would increase.

Equations (8.3) and (8.7) are also used to approximate changes in consumer and producer surplus that would result from a subsidy in the output market. Figure 8.5 presents the equilibria trajectories for a subsidy. In this case, the price received by producers increases to p_s^S and the price paid by consumers declines to p_s^D. Because $E(p^D) < 0$, $E(p^S) > 0$, and $E(q) > 0$, the change in consumer surplus (area a + area b) given by (8.3) and the change in producer surplus (area c + area d) given by (8.7) are both positive.

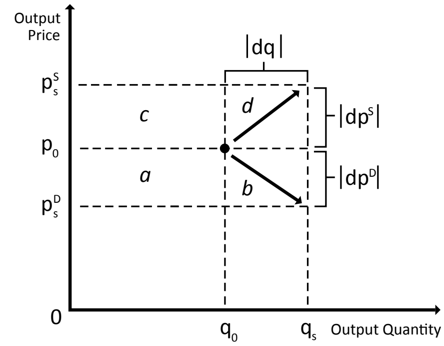

Figure 8.5. Equilibria Trajectories Resulting from a Producer Subsidy

Changes in Producer Surplus in Input Markets

Just, Hueth, and Schmitz (2004) note that changes in producer surplus caused by an exogenous shock can be estimated in the output market, as noted above, or estimated within input markets. Either approach can be used because an EDM traces out the equilibria trajectories in input markets that are caused by an exogenous shock to the output market. The left panel of Figure 8.6 reproduces the output market tax wedge and equilibria trajectories from Figure 8.4. The remaining two panels in Figure 8.6 present the equilibria trajectories for input markets 1 and 2 in the one-output, two-input example.

The equilibrium trajectory for input 1 in the middle panel of Figure 8.6 indicates that the tax on output causes the equilibrium price and quantity for input 1 ($w_{1,0}$, $x_{1,0}$) to decline to ($w_{1,t}$, $x_{1,t}$). The absolute values of the changes in input 1 price and quantity levels are denoted as $|dw_1|$ and $|dx_1|$, respectively.

The right panel in Figure 8.6 illustrates the effect of a tax in the output market on input 2. The equilibrium trajectory generated by the EDM indicates that the initial equilibrium input price and quantity ($w_{2,0}$, $x_{2,0}$) declines to ($w_{2,t}$, $x_{2,t}$). The absolute values of the change in input 2 price and quantity are denoted as $|dw_2|$

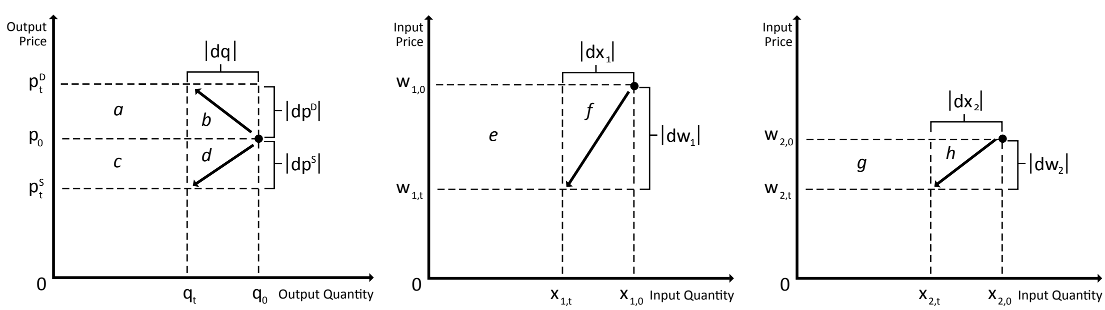

Figure 8.6. Effect of an Output Tax on the Output and Input Markets

and $|dx_2|$, respectively. Note that if only two inputs are used in a production process, the quantities of both inputs are reduced by the tax in the output market. However, if three or more inputs are considered along with variable input proportions technologies, then it is possible that the use of one of the inputs may increase even as the use of the others decline, as noted in Chapter 7 and Gardner (1988).

Following the procedures used to derive changes in producer surplus measured at the output level, the total change in producer surplus in input market 1 is negative (Figure 8.6). The reduction in surplus is given by area $(e + f)$ and, relative to the initial size of the input 1 market, is

(8.8) $$\Delta PS_{x_1} = E(w_1)\{1 + ½E(x_1)\}w_{1,0}x_{1,0}.$$

Figure 8.6 illustrates the change in producer surplus in input market 2, which is also negative. The size of the reduction is given by area $(g + h)$ and, relative to the initial size of input 2 market, is

(8.9) $$\Delta PS_{x_2} = E(w_2)\{1 + ½E(x_2)\}w_{2,0}x_{2,0}.$$

Just, Hueth, and Schmitz (2004) note that the sum of the changes in producer surplus in input markets is a direct measure of the total change in producer surplus as calculated in the output market. The connection is made by recalling that EDMs must be HD0 in all prices. In addition, entry and exit in competitive industries results in zero economic profits, such that the sum of all input expenditures is equal to the total revenue created by the industry. Thus, at the initial equilibrium,

(8.10) $$p_0q_0 = w_{1,0}x_{1,0} + w_{2,0}x_{2,0}.$$

Multiplying the right-hand side of (8.8) by $\frac{w_{1,0}x_{1,0}+w_{2,0}x_{2,0}}{w_{1,0}x_{1,0}+w_{2,0}x_{2,0}}$ results in

(8.11) $$\Delta PS_{x_1} = E(w_1)\{1 + ½E(x_1)\}\left[\frac{w_{1,0}x_{1,0}+w_{2,0}x_{2,0}}{w_{1,0}x_{1,0}+w_{2,0}x_{2,0}}\right]w_{1,0}x_{1,0}.$$

Using (8.10), expression (8.11) can be written as

(8.12) $$\Delta PS_{x_1} = [K_1E(w_1)\{1 + ½E(x_1)\}]p_0q_0,$$

where K_1 is the initial factor share of input 1.

Likewise, the change in producer surplus of input 2 is related to the output market by

(8.13) $$\Delta PS_{x_2} = [K_2E(w_2)\{1 + ½E(x_2)\}]p_0q_0\ ,$$

where K_2 is the initial factor share of input 2. The equations are valid for increases or decreases in the use of each input, but the policy shock that occurs when assigning positive or negative changes to surplus measures must be considered.

Deadweight Losses and the Incidence of a Tax

In addition to estimating relative changes in consumer and producer surplus that result from exogenous economic shocks, researchers are often interested in estimating deadweight losses that result from market interventions and their incidence on consumers and producers. This section develops these metrics for a tax imposed in the output market.

Consider (8.1), which can be written as

$$(8.14)\quad |\Delta CS| = (|dp^D| * q_0) - (½|dp^D||dq|)$$

or

$$(8.15)\quad |\Delta CS| = (|dp^D| * q_o) - (|dp^D||dq|) + (½|dp^D||dq|)$$

or

$$(8.16)\quad |\Delta CS| = \left[\left(E(p^D)\right)\{1 + E(q)\}\right]p_0q_0 - [½E(p^D)E(q)]p_0q_0.$$

The first term on the right-hand side of (8.16) represents the amount of the tax that is borne by consumers (area a in Figures 8.4 and 8.6), for which we use the acronym *CTAX*. The second term on the right-hand side of (8.16) represents consumers' share of deadweight loss (area b in Figures 8.4 and 8.6) caused by the tax, denoted as *CDWL*. Note that for a tax, $E(p^D) > 0$ and $E(q) < 0$, such that $CDWL > 0$. The tax causes $\Delta CS = -|\Delta CS|$ so that (8.16) is written as

$$(8.17)\quad \Delta CS = -\text{area } a - \text{area } b = -CTAX - CDWL.$$

A similar approach is used to develop an expression for the total change in producer surplus, where $PTAX_Q$ (area c in Figures 8.4 and 8.6) represents producers' share of the tax and $PDWL_Q$ (area d in Figures 8.4 and 8.6) is the producers' share of the deadweight loss generated by the tax:

$$(8.18)\quad \Delta PS_Q = [E(p^S)\{1 + E(q)\}]p_0q_0 - [½E(p^S)E(q)]p_0q_0$$

and

(8.19) $\Delta PS_Q = -\text{area } c - \text{area } d = -PTAX_Q - PDWL_Q.$

Note that changes in producer surplus in (8.19) are measured in the output market and designated with the subscript Q. Just, Hueth, and Schmitz (2004) show that the same calculation could be made in the input markets. Noting that $PTAX_Q > 0$ and $E(p^S) < 0$ such that $PTAX_Q = [-(E(p^S))\{1 + E(q)\}]p_0q_0$, we find that the change in producer surplus for input 1 is

(8.20) $$\Delta PS_{x_1} = -[(K_1E(w_1))\{1 + ½E(x_1)\}]p_0q_0$$
$$= -[(K_1E(w_1))\{1 + E(x_1)\}]p_0q_0 - [½K_1E(w_1)E(x_1)]\, p_0q_0$$

or

(8.21) $$\Delta PS_{x_1} = -PTAX_{x_1} - PDWL_{x_1},$$

where $PTAX_{x_1}$ = area e and $PDWL_{x_1}$ = area f.

The change in producer surplus for input 2 is given by

(8.22) $$\Delta PS_{x_2} = [(K_2E(w_2))\{1 + ½E(x_2)\}]p_0q_0$$
$$= [(K_2E(w_2))\{1 + E(x_2)\}]p_0q_0 - [½\, K_2E(w_2)E(x_2)]p_0q_0$$

or

(8.23) $$\Delta PS_{x_2} = -PTAX_{x_2} - PDWL_{x_2},$$

where $PTAX_{x_2}$ = area g and $PDWL_{x_2}$ = area h.

The values for $PTAX_{x_i}$ are positive if input prices and quantities decline but negative for increases in input prices and quantities. In this case, a tax on one input can be considered a negative tax or a subsidy on the second input.

Estimated changes in producer surplus using the output market in (8.19) should equal the sum of changes in producer surplus calculated for each of the input markets in (8.21) and (8.23) such that

(8.24) $$\Delta PS_Q = \Delta PS_{x_1} + \Delta PS_{x_2},$$

or area $(c + d) = (e + f) + (g + h)$. Just, Hueth, and Schmitz (2004) show that (8.24) holds for an underlying nonlinear system. However, the equivalency of the two approaches is not exact when using EDM linear approximations of such a system.

Changes in Producer and Consumer Surplus Caused by an Output Tax

We use a numerical example to illustrate the differences in producer surplus calculations when calculating changes in the output market versus the input markets. For the following numerical examples, we increase the accuracy of the results by reporting values to six decimal places to allow for more precise comparisons between the two methods.

Consider the one-output, two-input market presented in Chapter 5 with a 10% excise tax imposed in the output market. Our previous EDM results in (5.11) generated $E(q) = -0.031579$, $E(p^D) = 0.052632$, $E(p_S) = -0.047368$, $E(x_1) = -0.013158$, $E(x_2) = -0.039474$, $E(w_1) = -0.065789$, and $E(w_2) = -0.039474$. To generate these results, it was assumed that $K_1 = 0.30$ and $K_2 = 0.70$. Consequently, $K_1E(w_1) = -0.197367$, and $K_2E(w_2) = -0.27632$. For this example, we assume that the size of the industry, or its total revenue ($p_0 q_0$), is equal to \$1,000. Because of the excise tax, both consumer and producer surplus are reduced. From (8.3), the change in consumer surplus is calculated as

$$\Delta CS = -[0.052632\,\{1 + ½(-0.031579)\}] \times 1{,}000 \tag{8.25}$$

$$= [-0.051801] \times \$1{,}000 = -\$51.80.$$

Note that consumer surplus has declined by 5.18% relative to the initial \$1,000 of the industry's total revenue, or \$51.80 divided by \$1,000.

Next, consider the total change in producer surplus calculated in the output market using (8.7), which results in

$$\Delta PS_Q = [-0.047368\,\{1 + ½(-0.031579)\}] \times 1{,}000 \tag{8.26}$$

$$= [-0.046620] \times \$1{,}000 = -\$46.62.$$

The change in producer surplus calculated at the output market level represents a 4.66% decline from the initial size of the industry.

As noted above, an alternative method for calculating changes in producer surplus involves calculating individual changes in producer surplus for each input market. This approach would be used if researchers are interested in the incidence of surplus changes on each factor of production caused by an output tax. For input 1, (8.12) yields

$$\Delta PS_{x_1} = [0.30\,(-0.065789)\,\{1 + ½(-0.013158)\}] \times 1{,}000 \tag{8.27}$$

$$= [-0.01961] \times \$1{,}000 = -\$19.61.$$

The tax at the output market level reduced producer surplus of input 1 by 1.96% of the original output market size. One could also calculate percentage changes in surplus relative to the original size of the input market. We continue using the size of the output market as the base to facilitate comparisons between the two methods of calculating changes in producer surplus.

Likewise, (8.13) is used to estimate changes in surplus that occur to the producer of input 2:

$$\text{(8.28)} \quad \Delta PS_{x_2} = [0.70\ (-0.039474)\{1 + ½(-0.039474)\}] \times 1{,}000$$

$$= [-0.027086] \times \$1{,}000 = -\$27.09.$$

Producer surplus of input 2 has declined by 2.71% of the size of the output market. For comparison purposes, the process of calculating changes in producer surplus using (8.27) and (8.28) results in a measure of the loss of producer surplus of \$46.70. When calculated at the output level in (8.26), the estimated loss of producer surplus was \$46.62. The difference between the two is \$0.08 and is only 0.005% of the original amount of total revenue in the output market.

A Numerical Example of the Level and Incidence of an Output Tax

The level and incidence of a tax can be calculated using the market in which the price wedge was imposed. Hence, for an output tax, total tax receipts can be calculated using area $(a + c)$ in Figure 8.4, or

$$\text{(8.29)} \quad TAX_Q = CTAX + PTAX_Q.$$

The value for *CTAX* is obtained from the first term on the right-hand side of (8.16) as

$$\text{(8.30)} \quad CTAX = \left[\left(E(p^D)\right)\{1 + E(q)\}\right]p_0q_0$$

$$= [0.052632\{1 + (-0.031579)\}] \times \$1{,}000$$

$$= 0.05097 \times \$1{,}000 = \$50.97.$$

Thus, the consumer share of tax payments, *CTAX*, totals \$50.97.

The producer share of the tax, $PTAX_Q$, can also be estimated in the market in which the tax was imposed. Using the first term of the right-hand side of (8.18), $PTAX_Q$ is calculated as

$$(8.31) \quad PTAX_Q = \left[-\left(E(p^S)\right)\{1 + E(q)\}\right] p_0 q_0$$

$$= [0.047368\,\{1 + (-0.031579)\}] \times \$1{,}000$$

$$= 0.045873 \times \$1{,}000 = \$45.87.$$

Consequently, total tax receipts generated by a 10% tax on the output of the industry is given by the sum of (8.30) and (8.31) as

$$(8.32) \quad TAX_Q = CTAX + PTAX_Q = \$50.97 + \$45.87 = \$96.84.$$

Therefore, the tax generates 9.68% of the initial \$1,000 revenue of the industry.

The amount of tax paid by producers can also be calculated at the input level even though the tax was imposed at the output level. Note that summing the tax receipts, surplus, or deadweight losses from calculations made at the output market level and the input market level would double count the effect. Nonetheless, calculating producer tax payments at the input level allows for measuring the tax incidence on the producers of each input as opposed to only the total size of producer tax payments. The first term on the right-hand side of (8.20) provides the amount of tax paid by the producers of input 1, which is area *e* in Figure 8.6, or

$$(8.33) \quad PTAX_{x_1} = -\left[\left(K_1 E(w_1)\right)\{1 + E(x_1)\}\right] p_0 q_0$$

$$= -[(0.30)(-0.065789)\{1 + (-0.039474)\}] \times \$1{,}000$$

$$= 0.019477 \times \$1{,}000 = \$19.48.$$

The first term on the right-hand side of (8.22) provides the amount of tax paid by the producers of input 2, which is area g in Figure 8.6, or

$$(8.34) \quad PTAX_{x_2} = -\left[\left(K_2 E(w_2)\right)\{1 + E(x_2)\}\right] p_0 q_0$$

$$= -[(0.70)(-0.039474)\{1 + (-0.039474)\}] \times \$1{,}000$$

$$= 0.026541 \times \$1{,}000 = \$26.54.$$

Thus, the sum of (8.33) and (8.34) indicates that the total amount of tax paid by the producers of inputs 1 and 2, area $(e + g)$, equals \$46.02. Theoretically, the estimated sum of taxes paid by producers in the input markets (8.33 and 8.34) should equal the estimate of producer taxes calculated at the output level (8.31). However, the values differ by \$0.15 (\$46.02 – \$45.07). We show below that this difference can also be calculated by defining ***KEW*** = $[K_1E(w_1), K_2E(w_2), \ldots, K_nE(w_n)]'$ and $\boldsymbol{\Omega}$ =

$[\sigma_{ij}]$ as the matrix of Allen elasticities of substitution. Consequently, the difference between the two measures is given by

$$(8.35)\quad \boldsymbol{KEW}'\boldsymbol{\Omega}\,\boldsymbol{KEW} = [-0.197367\ \ -0.027632]\begin{bmatrix}-2.333333 & 1.00\\ 1.00 & -0.428571\end{bmatrix}\begin{bmatrix}-0.197367\\ -0.027632\end{bmatrix}$$

$$= \$0.15.$$

Thus, calculating the incidence of the tax at the input level generates an estimate of tax collections that is 0.33% larger (($46.02/$45.87) – 1) than that calculated at the output level. This difference is relatively small, and we conclude that the use of either method is reasonable for developing these estimates.[12]

A Numerical Example of the Incidence of Deadweight Losses Generated by an Output Tax

A tax imposed in the output market reduces economic activity and lowers the producer price of a good or service, while consumers pay a price for the good or service that is above its marginal cost of production. The combination of these two effects generates deadweight losses. Researchers are often interested in the total size of deadweight losses and their incidence among consumers and producers. These impacts can be calculated at the output level for a tax placed at that level. However, just as producer surplus and tax receipts can be calculated at either the output or input levels, deadweight losses can also be calculated at either level. The choice depends upon whether interest is in estimates of total deadweight producer losses or in the incidence of deadweight losses among input producers.

Equation (8.17) shows that the consumer portion of deadweight losses caused by the output tax, area *b* in Figure 8.4, is the second term of the right-hand side in (8.16), such that

$$(8.36)\quad CDWL = -[½E(p^D)E(q)]p_0q_0.$$

Using the above numerical example, the consumer portion of deadweight loss is calculated as

$$(8.37)\quad CDWL = -[½(0.052632)(-0.031579)] \times \$1{,}000$$

$$= 0.000831 \times \$1{,}000 = \$0.83.$$

12 This difference occurs because an EDM linearly approximates a nonlinear system. For fixed input proportion technologies, however, Allen elasticities of substitution are 0 and there would be no difference between the two approaches. Also, if the underlying system of equations were truly linear, then the Allen elasticities of substitution would approach infinity and there would again be no difference between the two approaches.

The tax placed in the output market, therefore, generated a deadweight loss to consumers of $0.83.

The producer component of deadweight losses can be calculated either at the output level or the input level. If the calculation is made at the output level, area *d* in Figure 8.4, the second term of (8.18) is used, such that

(8.38) $$PDWL_Q = [½E(p^S)E(q)]p_0q_0.$$

Using our numerical example, (8.38) becomes

(8.39) $$PDWL_Q = [½(-0.047368)(-0.03115791)] \times \$1{,}000$$

$$= [0.000748] \times \$1{,}000 = \$0.75.$$

Hence, total producer deadweight loss calculated at the output level is $0.75. Adding *CDWL* to $PDWL_Q$ generates a total deadweight loss from the tax imposed at the output market of $1.58 ($0.83 + $0.75), which represents 0.16% of the initial total revenue of the industry.

Producer deadweight losses can also be calculated at the input level if one desires to measure the incidence of producer deadweight losses among the input markets. In this case, the share of deadweight loss, area *f* in Figure 8.6, borne by the producer of input 1 ($PDWL_{x_1}$) is provided by the second term of the right-hand side of (8.20), such that

(8.40) $$PDWL_{x_1} = [½K_1E(w_1)E(x_1)]p_0q_0.$$

Numerically, (8.40) becomes

(8.41) $$PDWL_{x_1} = [½(0.30)(-0.065789)(-0.013158)] \times \$1{,}000$$

$$= [0.000130] \times \$1{,}000 = \$0.13.$$

Therefore, the producer of input 1 incurs $0.13 of deadweight loss. Likewise, the producer of input 2 incurs a deadweight loss, area *h* in Figure 8.6, estimated by the second term of the right-hand side of (8.22), such that

(8.42) $$PDWL_{x_2} = [½K_2E(w_2)E(x_2)]p_0q_0.$$

Numerically, (8.42) becomes

$$(8.43) \quad PDWL_{x_2} = [½(0.70)(-0.039474)(-0.039474)] \times \$1{,}000$$

$$= [0.000545] \times \$1{,}000 = \$0.55.$$

Hence, the producer of input 2 incurs a deadweight loss of \$0.55. The total *PDWL* as calculated in the input markets is the sum of (8.42) and (8.43), or \$0.68.

Note that total producer deadweight loss calculated at the output level is \$0.07 larger than that calculated at the input level. The quadratic expression that accounts for this difference is given by

$$(8.44) \quad ½\boldsymbol{KEW}'\,\boldsymbol{\Omega}\,\boldsymbol{KEW} = ½[-0.197368 \;\; -0.027632]\begin{bmatrix}-2.3333 & 1.00\\ 1.00 & -0.42867\end{bmatrix}\begin{bmatrix}-0.197368\\ -0.027632\end{bmatrix} =$$

$$= -\$0.07.$$

The difference between the two approaches is relatively small.

An Application to the Beef Market

Equations (5.2)–(5.8) present an EDM that allowed for the estimation of the impacts of a 10% excise (or sales) tax on output. We estimated changes and incidence of surplus, tax receipts, and deadweight losses caused by the tax for a hypothetical market. However, suppose the product of interest is retail beef products for which (5.11) indicates that a 10% excise tax on beef production would reduce output by 3.2%. Assuming an initial equilibrium beef consumption quantity of 60 pounds per capita, as illustrated in Figure 8.7, the EDM calculates the equilibrium trajectory caused by the excise tax. The 3.2% reduction in output from the initial equilibrium represents a decline of 1.89 pounds per capita resulting in a new equilibrium quantity of 58.11 pounds per capita.

In addition, the excise tax drives a wedge between consumer and producer prices, as noted in (5.11). The consumer price of beef, $E(p^D)$, increases by 5.3%. Using \$5.00 per pound as the initial equilibrium price, the new equilibrium price that consumers pay is \$5.00 ∗ (1 + 0.053) or \$5.27 per pound. In addition, (5.11) indicates that producer prices, $E(p^S)$, fall by 4.7%, such that the new equilibrium price received by producers is \$4.77 per pound, or \$5.00 ∗ (1 – 0.047).

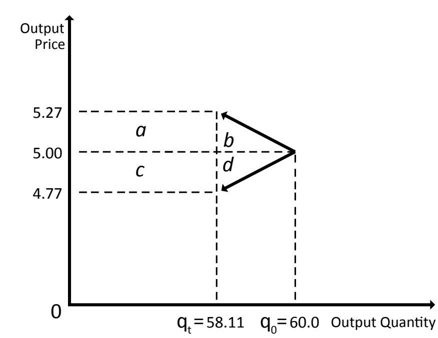

Figure 8.7. Consumer and Producer Surplus and Deadweight Loss from an Excise Tax

Note that the 10% tax generates a wedge between consumer and producer prices of $0.50 per pound, which is 10% of the initial equilibrium price. Equation (5.11) indicates that the implied own-price elasticity of supply, $E(q)/E(p^S)$, is equal to (–0.032/–0.047), or 0.68. Consequently, consumers bear a larger portion of the excise tax relative to producers because the assumed own-price elasticity of beef demand (–0.60) is more inelastic than the EDM-implied own-price elasticity of beef supply (0.68).

The price and quantities noted above and in (8.3) are used to estimate the change in consumer surplus caused by the excise tax. The calculation measures area $(a + b)$ in Figure 8.7:

(8.45) $$\Delta CS = -\left[\left(E(p^D)\right)\{1 + ½E(q)\}\right]p_0 q_0$$

$$= [-0.052332\{1 + ½(-0.0315679)\}](\$5.00 \times 60).$$

$$= [-0.051801] \times \$300$$

$$= -\$15.54.$$

An average beef consumer has lost $15.54 of consumer surplus, which is 5.2% of their total expenditures of $300 (60 pounds per capita * $5.00 per pound on beef prior to the excise tax). The estimated change is easily expanded to include the entire beef market by multiplying the result by the U.S. population.

The excise tax reduces producer surplus, which is calculated at the output level using (8.7) and the results from (5.11). Specifically, the impact of the 10% excise tax on producer surplus is calculated as area $(c + d)$ in Figure 8.7:

(8.46) $$\Delta PS_Q = [-0.047868\,\{1 + ½(-0.031579)\}](\$5.00 \times 60)$$

$$= [-0.046621] \times \$300$$

$$= -\$13.99.$$

Producer surplus has declined by $13.99, or 4.7% of the initial $300 per capita expenditures on beef. The change in producer surplus could be calculated at the input levels, and the result would be only slightly different from that in (8.46).

The total amount of excise tax receipts is calculated using (8.29), which indicates that both consumers and producers pay some of the tax. The per capita consumer share of the tax is represented by area a in Figure 8.7, and is calculated using (8.30):

(8.47) $$CTAX = \left[\left(E(p^D)\right)\{1 + E(q)\}\right]p_0 q_0$$

$$= [0.052632\,\{1 + (-0.0315791)\}](\$5.00 \times 60)$$

$$= [0.050970] \times \$300 = \$15.29.$$

The amount of tax paid by producers, area *c* in Figure 8.7, is calculated using (8.31):

(8.48) $$PTAX_Q = \left[-\left(E(p^S)\right)\{1 + E(q)\}\right]p_0 q_0$$

$$= [0.047368\,\{1 + (-0.0315791)\}](\$5.00 \times 60)$$

$$= [0.045873] \times \$300 = \$13.76.$$

The producers' share of the tax is $13.76 per capita, and the combination of consumer and producer tax receipts sums to $29.05 per capita ($15.29 + $13.76).

The deadweight loss caused by the excise tax consists of a consumer component and a producer component. Consumer deadweight losses are represented by area *b* in Figure 8.7 and calculated using (8.36):

(8.49) $$CDWL = [-½E(p^D)E(q)]p_0 q_0.$$

$$= [-½(0.0526321)(-0.0315791)](\$5.00 \times 60)$$

$$= [0.000831] \times \$300 = \$0.25.$$

Producer deadweight losses are represented by area *d* in Figure 8.7 and calculated at the output level using (8.38):

(8.50) $$PDWL_Q = [½E(p^S)E(q)]p_0 q_0$$

$$= [½(-0.047368)(-0.0315791)](\$5.00 \times 60)$$

$$= [0.000748] \times \$300 = \$0.22.$$

Summing (8.49) and (8.50) results in a total deadweight loss estimate of $0.47 per capita.

Note that total deadweight losses are also equal to the sum of the change in consumer surplus, the change in producer surplus, and total tax receipts per capita:

(8.51) $DWL_Q = \Delta CS + \Delta PS_Q + \Delta TAX_Q.$

In our numerical application, (8.51) is

(8.52) $DWL_Q = -15.54 - 13.99 + 29.05 = \$0.47$

which is the same value as that obtained by summing (8.49) and (8.50).

Difference between the Two Approaches for Calculating Changes in Producer Surplus

As noted earlier, we have developed an expression that compares the calculations of producer surplus using the output market to those obtained by summing producer surplus changes across *n* input markets. The comparison is developed by first substituting (4.26) into (8.7) to obtain a measure of producer surplus changes at the output level:

(8.53) $$\Delta PS_Q = \left[\left(E(p^S)\right)\{1 + ½E(q)\}\right]p_o q_o$$
$$= [(\textstyle\sum_{i=1}^{n} K_i E(w_i))\{1 + ½[E(q)\}]p_0 q_0.$$

To calculate changes in producer surplus at the input level, PS_x, (8.11) and (4.28) are substituted into (8.24):

(8.54) $$\Delta PS_X = \textstyle\sum_{i=1}^{n} \Delta PS_{x_i}$$
$$= \left[(\textstyle\sum_{i=1}^{n} K_i E(w_i))\left\{1 + ½\left(E(q) + \sum_{j=1}^{n} \sigma_{ij} K_j E\left(w_j\right)\right)\right\}\right] p_0 q_0.$$

The difference between (8.53) and (8.54) in the multiple input case becomes

(8.55) $$\Delta PS_X - \Delta PS_Q = ½ \textstyle\sum_{i=1}^{n} \sum_{j=1}^{n} K_i E(w_i)\sigma_{ij} K_j E\left(w_j\right)$$
$$= ½\, \boldsymbol{KEW'}\, \boldsymbol{\Omega}\, \boldsymbol{KEW}.$$

Surplus and Deadweight Losses from a Tax on an Input

Imposing an excise tax on a input drives a wedge between the input's demand and supply price. In addition, the use of that input and the output produced by the production process declines. If only two inputs are used in the production process, then the use of both decline. However, if more than two inputs are involved in the production of an output, then the use of the taxed input declines but it is possible for the use of other inputs to increase even though overall output is always reduced. For example, Gardner's (1988, p. 114) three-input example shows that imposing a tax on input 1 causes a reduction in the use of that input as well as the use of input 2. However, the use of input 3 increases, even though total output is reduced. We present this outcome using stylized equilibrium trajectory plots.

The upper right panel of Figure 8.8 shows the equilibrium trajectories for the price and use of input 1 after the imposition of a tax on that input. Note that the tax places a wedge between the input supply price, $w^{S}_{1,t}$, and the input demand price, $w^{D}_{1,t}$. In the output market (the upper left panel of Figure 8.8), the tax on input 1 causes the quantity of output to decline while increasing the price of the output. The price of output increases from p_0 to p_t, while the quantity consumed in the output market declines from q_0 to q_t.

The tax on input 1 causes the amount of input 1 used in the production process to decline from $x_{1,0}$ to $x_{1,t}$, as shown in the upper right panel of Figure 8.8. The tax on input 1 in Gardner's example also caused the price and use of input 2 to decline. The EDM provides the equilibrium trajectory for this decline, as illustrated by the arrow in the lower left panel in Figure 8.8. The figure indicates that the tax on input 1 results in the price of input 2 declining from $w_{2,0}$ to $w_{2,t}$ and the use of the input declining from $x_{2,0}$ to $x_{2,t}$.

Conversely, the tax on input 1 increases the use and price of input 3. The lower right panel in Figure 8.8 shows the equilibrium trajectory that is estimated by the

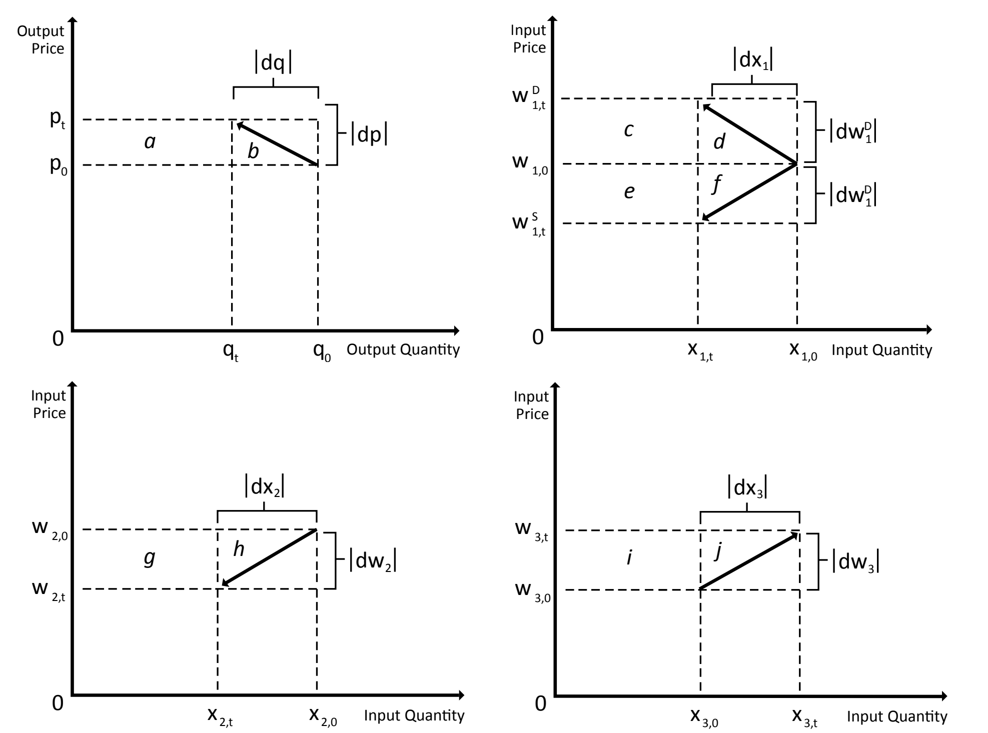

Figure 8.8. A Tax on Input 1

EDM for input 3. The price of input 3 increases from $w_{3,0}$ to $w_{3,t}$, and the use of the input increases from $x_{3,0}$ to $x_{3,t}$.

Changes in Consumer Surplus, Producer Surplus, Tax Incidence, and Deadweight Losses from a Tax on an Input

The tax wedge imposed in the input 1 market causes changes in producer and consumer surplus, deadweight losses, and generates tax receipts. The change in consumer surplus is shown as area $(a + b)$ in Figure 8.8 and calculated in the output market using the EDM equilibrium trajectories. However, changes and their incidence in producer surplus, deadweight losses, and tax receipts must be calculated at the input level because there is no mechanism in the output market to facilitate those calculations when a tax is imposed on an input.

Using the Output Market to Measure the Effects of a Tax on an Input

Consider changes in consumer surplus as indicated in Figure 8.8, where areas a and b represent a loss of consumer surplus and can be calculated using (8.3):

$$\Delta CS = -\text{area}\ (a+b) = -\left[\left(E(p^D)\right)\{1 + ½E(q)\}\right]p_0 q_0. \tag{8.56}$$

Note that part of this loss, area a, is in the form of taxes effectively paid by consumers. Thus $CTAX$ is calculated as shown in (8.30):

$$CTAX = \text{area}\ (a) = \left[\left(E(p^D)\right)\{1 + E(q)\}\right]p_0 q_0. \tag{8.57}$$

The $CDWL$ (area b) is the difference between the change in consumer surplus shown in (8.56) and the amount of the tax paid by consumers (8.57):

$$CDWL = -[½E(p^D)E(q)]p_0 q_0. \tag{8.58}$$

Measuring Effects in the Input Markets of a Tax on an Input

Changes in producer surplus caused by a tax on an input must be measured in the input markets. Although an EDM provides the necessary results to calculate changes in consumer surplus caused by a tax in the input markets, it does not provide a mechanism for measuring changes in producer surplus at the output level.

When a tax is placed on an input, changes in producer surplus are calculated by measuring the effects across all input markets. The change in producer surplus in input market 1 is given by the negative of area $(e + f)$ in Figure 8.8 and is calculated as

(8.59) $$\Delta PS_{x_1} = -\text{area } (e+f) = [K_1 E(w_1^S)\{1 + ½E(x_1)\}]p_0 q_0$$

$$= [E(w_1^S)\{1 + ½E(x_1)\}]w_{1,0}x_{1,0},$$

using the perfectly competetive assumption that $K_1 p_0 q_0 = w_{1,0}x_{1,0}$. Because the tax reduces the use of input 1, the change in producer surplus is negative. Likewise, the change in producer surplus of input 2 caused by the imposition of a tax on input 1 is represented by the negative of area $(g + h)$ in Figure 8.8 and calculated as

(8.60) $$\Delta PS_{x_2} = -\text{area } (g+h) = [K_2 E(w_2)\{1 + ½E(x_2)\}]p_0 q_0$$

$$= [E(w_2)\{1 + ½E(x_2)\}]w_{2,0}x_{2,0}.$$

This value is negative because a smaller amount of input 2 is used and the price of the input is lowered because of the tax.

However, in this example, the use of input 3 increases as a result of the tax on input 1. The suppliers of input 3 experience an increase in producer surplus of area $(i + j)$, which is calculated as

(8.61) $$\Delta PS_{x_3} = \text{area } (i+j) = [K_3 E(w_3)\{1 + ½E(x_3)\}]p_0 q_0$$

$$= [E(w_3)\{1 + ½E(x_3)\}]w_{3,0}x_{3,0}.$$

Note that this positive value indicates that the tax imposed on input 1 essentially subsidizes the producers of input 3. That is, the tax on input 1 has caused more of input 3 to be used. The sum of (8.59), (8.60), and (8.61) is the net loss of producer surplus, $(i + j)$ – area $(e + f + g + h)$, or

(8.62) $$\Delta PS_X = \Delta PS_{x_1} + \Delta PS_{x_2} + \Delta PS_{x_3}.$$

Deadweight Losses in the Input Markets from a Tax on an Input

The imposition of a tax on an input requires that producer deadweight losses be calculated in the input markets, although consumer deadweight losses continue to be calculated in the output market. The deadweight loss in input market 1 is given by

(8.63) $$PDWL_{x_1} = \text{area } (f) = [½K_1 E(w_1^S)E(x_1)]p_0 q_0$$

$$= [½E(w_1^S)E(x_1)]w_{1,0}x_{1,0}.$$

Similarly, the deadweight loss incurred by the producers of input 2 is given by

$$(8.64)\quad PDWL_{x_2} = \text{area }(h) = [\tfrac{1}{2}\, K_2 E(w_2)E(x_2)]p_0 q_0$$

$$= [\tfrac{1}{2}\, E(w_2)E(x_2)]w_{2,0}x_{2,0}.$$

While the producers of input 3 benefit from the tax on input 1, societal deadweight losses occur in the input 3 market (area *j*). We denote this as

$$(8.65)\quad PDWL_{x_3} = \text{area }(j) = [\tfrac{1}{2}\, K_3 E(w_3)E(x_3)]p_0 q_0$$

$$= [\tfrac{1}{2}\, E(w_3)E(x_3)]w_{3,0}x_{3,0}.$$

Incidence of a Tax on an Input Measured in the Input Markets

Although a tax was placed on input 1, consumers share in the payment of this tax. The amount of the tax borne by consumers is given by (8.57) and is calculated at the output level. However, because the tax was placed on an input, the producers' share of the tax can only be calculated at the input level. The amount of the excise tax paid by the producer of input 1 is given by

$$(8.66)\quad PTAX_{x_1} = \text{area }(e) = [-K_1 E(w_1^S)\{1 + E(x_1)\}]p_0 q_0$$

$$= [-E(w_1^S)\{1 + E(x_1)\}]w_{1,0}x_{1,0}.$$

Although the tax is applied on input 1, the producers of the second input incur some of the burden of the tax, which is calculated as

$$(8.67)\quad PTAX_{x_2} = \text{area }(g) = [-K_2 E(w_2)\{1 + E(x_2)\}]p_0 q_0$$

$$= [-E(w_2)\{1 + E(x_2)\}]w_{2,0}x_{2,0}.$$

Conversely, the producers of input 3 receive, rather than pay, a portion of the tax paid by others because both the price and the quantity supplied of input 3 increase. The size of this gain, which is essentially a negative tax, is provided by area *i*:

$$(8.68)\quad PTAX_{x_3} = -\text{area }(i) = -[-K_3 E(w_3)\{1 + E(x_3)\}]p_0 q_0$$

$$= -[-E(w_3)\{1 + E(x_3)\}]w_{3,0}x_{3,0}.$$

Total tax revenues paid to the government that result from the tax on input 1 are equal to the sum of taxes paid by consumers and producers:

$$(8.69)\quad TTAX = \text{area } (a + e + g - i)$$
$$= CTAX + PTAX_{x_1} + PTAX_{x_2} + PTAX_{x_3}.$$

Derived Demand and the Marginal Value of an Input

The previous sections described the measurement of changes in consumer and producer surplus, tax receipts, and deadweight losses when a tax is placed on an input. Interestingly, aggregate consumer and producer surplus, tax receipts, and deadweight losses can also be calculated using only the information contained in the input market being taxed. In some cases, one may only be interested in aggregate surplus, tax, and deadweight loss estimates of an exogenous policy that imposes a price wedge in an input market. In addition, it could be that several input markets are being modeled, and a researcher might not be particularly interested in the incidence of the tax on each.

In the preceding example, the excise placed on input 1 is illustrated in the upper right panel of Figure 8.8. The impact on producer surplus for input 1 is indicated by area $(e + f)$. Area $(c + d)$ contains the aggregate measure of consumer surplus plus the net changes in producer surplus that occur in input markets 2 and 3. Consequently, the aggregate change in surplus can be obtained by summing area $(c + d)$ and area $(e + f)$.

We define area $(c + d)$ as the derived demand surplus (DDS) of the economic system illustrated in Figure 8.8. We use this term because we are measuring surplus changes in the derived demand input sector of the market in which the tax was placed. The DDS, area $(c + d)$ in Figure 8.8, is subject to the previously discussed small EDM curvature errors. Area $(c + d)$ approximately equals the sum of the output market consumer surplus loss shown as area $(a + b)$ plus the input 2 producer surplus loss in area $(g + h)$, less the increase in input 3 producer surplus, which is area $(i + j)$. Thus, area $(c + d)$ in the upper right panel of Figure 8.8 represents the change in consumer surplus plus the net changes in producer surplus that occur in input markets 2 and 3 resulting from the tax on input 1. In addition, if area $(c + d)$ is added to area $(e + f)$, the sum provides an estimate of the total net change in surplus within the entire system caused by the tax on input 1. This outcome occurs because area $(e + f)$ represents the loss of producer surplus in input market 1 while area $(c + d)$ provides an estimate of the total net change in surplus in the other input and output markets included in the EDM. Therefore, the derived demand price, $w^{D}_{1,t}$, represents the marginal value of input 1 with respect to the entire system. We show that area $(c + d)$ is equivalent to the sum of area

$(a + b)$ and area $(g + h)$ less area $(i + j)$ within a small approximation error caused by the inability of EDMs to model curvature.

Derived Demand Surplus and Changes in Total Surplus from a Tax on an Input

Using similar procedures to those of previous derivations, the derived demand surplus calculation at the input level is given by

(8.70) $$\Delta DDS_{x_1} = -\text{area}\ (c + d) = [-K_1 E(w_1^D)\{1 + ½E(x_1)\}]p_0 q_0$$
$$= [E(w_1^D)\{1 + ½E(x_1)\}]w_{1,0}x_{1,0}.$$

As noted above, this area is, within a generally small approximation error, equivalent to the area $(a + b + g + h - i - j)$:

(8.71) $$\Delta DDS_{x_1} \approx -\text{area}\ [(a + b) + (g + h) - (i + j)]$$
$$= \Delta CS + \textstyle\sum_{i=2}^{3} \Delta PS_i.$$

To illustrate, we derive an expression for the difference between (8.70) and (8.71). We use the result that $E(w_1^D) = E(w_1^S)$ for $i = 2, \dots, n$ so that (8.71) becomes

(8.72) $$\Delta DDS_{x_1} = [-K_1 E(w_1^D)\{1 + ½E(x_1)\}]p_0 q_0$$
$$= \left[-K_1 E(w_1^D)\{1 + (½E(q) + \textstyle\sum_{j=1}^{3} K_j \sigma_{ij} E\left(w_j^D\right))\}\right]p_0 q_0$$

or

(8.73) $$\Delta DDS_{x_1} = [-K_1 E(w_1^D)\{1 + ½E(q)\}$$
$$- ½K_1 E(w_1^D) \textstyle\sum_{j=1}^{3} K_j \sigma_{ij} E\left(w_j^D\right)]p_0 q_0.$$

Noting that $E(p^D) = E(p^S)$ and using the results from (8.3), (8.12), and (8.13), the right-hand side of (8.71) can be written as

(8.74) $$[-E(p_S)\{1 + ½\ E(q)\}]$$
$$+ ½ \textstyle\sum_{i=2}^{3}[K_i E(w_i^D)\{1 + ½E(x_i)\}]p_0 q_0.$$

We note that

$$(8.75)\quad E(p^S) = K_1E(w_1^D) + \textstyle\sum_{i=2}^{3} K_iE(w_i^D)$$

and

$$(8.76)\quad E(x_i) = E(q) + \textstyle\sum_{j=1}^{3} K_j\sigma_{ij}E\big(w_j^D\big).$$

Substituting (8.74) and (8.75) into (8.76) results in a function that represents the right-hand side of (8.72):

$$(8.77)\quad \big[-K_1E(w_1^D)\{1 + ½E(q)\} + ½\textstyle\sum_{i=2}^{3} K_iE(w_i^D)\sum_{j=1}^{3} K_j\sigma_{ij}E\big(w_j^D\big)\big]p_0q_0.$$

Subtracting (8.77) from (8.72) yields

$$(8.78)\quad -½\textstyle\sum_{i=1}^{3} K_iE(w_i^D)\sum_{j=1}^{3} K_j\sigma_{ij}E\big(w_j^D\big)$$

or

$$(8.79)\quad -½\textstyle\sum_{i=1}^{3}\sum_{j=1}^{3} K_jE(w_i^D)\sigma_{ij}K_iE(w_i^D)$$

or

$$(8.80)\quad -½\boldsymbol{KEW'\Omega KEW}.$$

so that

$$(8.81)\quad \Delta DDS_{x_1} - (\Delta CS + \textstyle\sum_{i=2}^{3}\Delta PS_i) \approx -½\boldsymbol{KEW'\Omega KEW}.$$

Derived Demand and the Incidence of a Tax on Input 1

Total tax receipts from all market participants can be calculated by considering only the input market on which the tax was imposed. Area *c* in Figure 8.8 is almost equivalent to the tax paid by consumers, area *a* in Figure 8.8, plus the tax paid by the producer of input 2, area *g*, less the subsidy gained by the producer of input 3, area *i*. Hence, total tax receipts can be calculated by summing area *c* with the taxes paid by the producer of input 1 (area *e*).

To illustrate, we define derived demand tax receipts (*DDTAX*) as

(8.82) $$DDTAX_{x_1} = \text{area }(c) = [K_1 E(w_1^D)\{1 + E(x_1)\}]p_0 q_0$$

$$= [E(w_1^D)\{1 + E(x_1)\}]w_{1,0}x_{1,0}.$$

The term derived demand is used to indicate that tax receipts are being calculated in the input market in which the tax was imposed. This value is approximately equivalent to *CTAX* in (8.57) and the sum of taxes, including negative taxes, paid by $PTAX_{x_2}$ and the input 3 market, $PTAX_{x_3}$, as shown in (8.67) and (8.68):[13]

(8.83) $$DDTAX_{x_1} \approx \text{area }(a + g - i) = \left(CTAX + \sum_{i=2}^{3} PTAX_{x_i}\right).$$

The values calculated in (8.82) and (8.83) are not identical, and the generally small difference between the two is equal to

(8.84) $$DDTAX_{x_1} - \left(CTAX + \sum_{i=2}^{3} PTAX_{x_i}\right) \approx \boldsymbol{KEW'\Omega KEW}.$$

Given that this error is usually small, total tax receipts can be calculated using only the market in which the tax was imposed:

(8.85) $$TTAX_{x_1} = \text{area }(c + e) = DDTAX_{x_1} + PTAX_{x_1}.$$

Derived Demand and Deadweight Losses from a Tax on Input 1

The total deadweight loss across all market participants can be calculated by considering only the input market on which the tax was imposed. Area *d* in Figure 8.8 is approximately equivalent to the deadweight loss incurred by consumers (area *b*) in Figure 8.8 and area (*h* + *j*), with total deadweight losses calculated by summing area *d* with area *f*.

To illustrate, using procedures similar to those in expressions (8.8)–(8.12), we define derived demand deadweight loss (*DDDWL*) as

(8.86) $$DDDWL_{x_1} = \text{area }(d) = [-½K_1 E(w_1^D)E(x_1)]p_0 q_0$$

$$= [-½E(w_1^D)E(x_1)]w_{1,0}x_{1,0}$$

with the term "derived demand" used to indicate that the calculation is being made at the input level. This value is approximately equivalent to *CDWL* in (8.58) and the sum of deadweight losses in the input 2 market, $PDWL_{x_2}$, from (8.63) and the input 3 market, $PDWL_{x_3}$:

13 Note that $PTAX_{x_i}$ can be negative if the equilibrium quantity and price increase for input *i*.

(8.87) $DDDWL_{x_1} \approx \text{area } (b + h + j) = CDWL + \sum_{i=2}^{3} PDWL_{x_i}.$

The values calculated in (8.86) and (8.87) are not identical, but the difference between the two is given by:

(8.88) $DDDWL_{x_1} - \left(CDWL + \sum_{i=2}^{3} PDWL_{x_i}\right) \approx -\frac{1}{2}\boldsymbol{KEW'\Omega KEW}.$

Given that this error is usually small, total deadweight losses can be calculated as

(8.89) $DWL = \text{area } (d + f) = DDDWL_{x_1} + PDWL_{x_1}.$

Thus, aggregate system deadweight losses can be calculated using only the changes in the input market that incurred the tax.

A Numerical Example of an Input Tax

We use the one-output, three-input EDM example from (7.79)–(7.94) and impose a 10% excise tax on input 1. For the following numerical example, we again use values to the sixth decimal place to improve precision. The tax is entered in vector $\boldsymbol{b}$ of (7.96) as $E(\theta_{14}) = 0.10$ with all other values of the vector set equal to 0. The $\boldsymbol{A}$ matrix in (7.96) is populated using an own-price elasticity of demand, η^D, of –0.40 while the own-price elasticities of input supply (ε_1, ε_2, ε_3) are 2.0, 1.0, and 0.20, respectively, and the factor shares of x_1 (K_1), x_2 (K_2), and x_3 (K_3) are 0.20, 0.50, and 0.30. The Allen elasticities of substitution (AES) are $\sigma_{12} = \sigma_{21} = 0.30$, $\sigma_{13} = \sigma_{31} = 1.0$, $\sigma_{23} = \sigma_{32} = 0.50$. Hence,

$$\sigma_{11} = -\frac{(K_2\sigma_{12}) + (K_3\sigma_{13})}{K_1} = -2.25$$

$$\sigma_{22} = -\frac{(K_1\sigma_{21}) + (K_3\sigma_{23})}{K_2} = -0.42$$

$$\sigma_{33} = -\frac{(K_1\sigma_{31}) + (K_2\sigma_{32})}{K_3} = -1.50.$$

The solution to (7.96) yields the following changes in the endogenous variables:

$$\text{(8.90)}\quad \boldsymbol{y} = \begin{bmatrix} E(q^D) \\ E(p^D) \\ E(q^S) \\ E(p^S) \\ E(x_1^D) \\ E(x_2^D) \\ E(x_3^D) \\ E(w_1^D) \\ E(w_2^D) \\ E(w_3^D) \\ E(x_1^S) \\ E(x_2^S) \\ E(x_3^S) \\ E(w_1^S) \\ E(w_2^S) \\ E(w_3^S) \end{bmatrix} = \begin{bmatrix} -0.007710 \\ 0.019276 \\ -0.007710 \\ 0.019276 \\ -0.040098 \\ -0.000870 \\ 0.002481 \\ 0.079951 \\ -0.000870 \\ 0.012403 \\ -0.040098 \\ -0.000870 \\ 0.002481 \\ -0.020049 \\ -0.000870 \\ 0.012403 \end{bmatrix}.$$

The 10% tax placed on input 1 causes the demand output price, $E(p^D)$, and supply price, $E(p^S)$, to increase by 1.93% as the demand quantity, $E(q^D)$, and supply quantity, $E(q^S)$, decline by 0.77%. The amount of input 1 used in the production process, $E(x_1^D)$ and $E(x_1^S)$, declines by 4.01%. Note that the demand price of input 1, $E(w_1^D)$, has increased by 8.0% while its supply price, $E(w_1^S)$, has declined by 2.0%. The sum of the absolute values of these changes is equal to the 10% tax imposed on the input 1 market. Because of the decline in output, the amount of input 2, $E(x_2^D)$ and $E(x_2^S)$, declines by 0.09% and its price declines by the same amount. Although output has declined, the use of input 3, $E(x_3^D)$ and $E(x_3^S)$, increases by 0.25% while its price, $E(w_3^D)$ and $E(w_3^S)$, increases by 1.24%.

Measuring Changes in Consumer Surplus, Tax Receipts, and Deadweight Loss at the Output Level

We assume that the initial total revenue of the industry (p_0q_0) is equal to \$1,000. Using the above values, the change in consumer surplus is given by

$$\text{(8.91)}\quad \Delta CS = -\text{ area }(a+b) = -\left[\left(E(p^D)\right)\{1 + ½E(q)\}\right]p_0q_0$$

$$= -[0.019276\,\{1 + ½(-0.007710)\}] \times \$1{,}000$$

$$= -0.019202 \times \$1{,}000 = -\$19.20.$$

The amount of the tax paid by consumers is

$$(8.92) \quad CTAX = \text{area } (a) = \left[\left(E(p^D)\right)\{1 + E(q)\}\right] p_0 q_0$$

$$= [0.019276\,\{1 + (-0.007710)\}] \times \$1{,}000$$

$$= 0.019128 \times \$1{,}000 = \$19.13.$$

The consumer share of the deadweight loss caused by the tax is calculated as

$$(8.93) \quad CDWL = \text{area } (b) = [-½E(p^D)E(q)]p_0 q_0.$$

$$= [-½((0.019276)(-0.007710))] \times \$1{,}000$$

$$= 0.000074 \times \$1{,}000 = \$0.07.$$

Note that changes in producer surplus, producer deadweight losses, and taxes paid by producers cannot be calculated in the output market because the tax and the resulting price wedge occurred in an input market.

Measuring Changes in Producer Surplus at the Input Level Caused by a Tax on an Input

Changes in producer surplus are calculated at the input level when a tax is imposed in the input markets. Losses of producer surplus are represented by areas e and f in input market 1, areas g and h in input market 2, and surplus gains in input market 3 as indicated by areas i and j (Figure 8.8). Thus, the total net change in producer surplus is given by

$$(8.94) \quad \Delta PS_X = \Delta PS_{x_1} + \Delta PS_{x_2} + \Delta PS_{x_3}.$$

Using (8.59), (8.60), and (8.61), expression (8.94) becomes

(8.95) $$\Delta PS_X = \{[(0.2)(-0.012403)\{1 + ½(-0.040098)\}] \times \$1{,}000\}$$
$$+ \{[(0.5)(-0.000870)\{1 + ½(-0.000870)\}] \times \$1{,}000\}$$
$$+ \{[(0.3)(0.012403)\{1 + ½(0.002481)\}] \times \$1{,}000\}$$
$$= (-0.003929 \times \$1{,}000) + (-0.000435 \times \$1{,}000)$$
$$+ (0.003726 \times \$1{,}000)$$
$$= -\$3.93 - \$0.43 + \$3.73 = -\$0.64.$$

Tax Receipts Paid at the Input Level Caused by a Tax on an Input

Total net tax receipts paid by the three input markets are calculated by summing the net taxes paid by each:

(8.96) $$PTAX_X = PTAX_{x_1} + PTAX_{x_2} + PTAX_{x_3}.$$

Note that the third input market is not incurring a tax payment but is the recipient of a tax "subsidy" given that the output and price of input 3 has increased. Numerically, (8.96) becomes

(8.97) $$PTAX_X = \{[-(0.2)(-0.020049)\{1 + (-0.040098)\}] \times \$1{,}000\}$$
$$+ \{[-(0.5)(-0.000870)\{1 + (-0.000870)\}] \times \$1{,}000\}$$
$$+ \{[-(0.3)(0.012403)\{1 + 0.002481\}] \times \$1{,}000\}$$
$$= (0.003849 \times \$1{,}000) + (0.000435 \times \$1{,}000)$$
$$+ (-0.003730 \times \$1{,}000)$$
$$= \$3.85 + \$0.43 - \$3.73 = \$0.55.$$

Deadweight Losses at the Input Level Caused by a Tax on an Input

Total net deadweight losses at the factor level are obtained by summing the net deadweight losses incurred by each of the three inputs:

(8.98) $$PDWL_X = PDWL_{x_1} + PDWL_{x_2} + PDWL_{x_3}.$$

Numerically, (8.98) becomes

$$(8.99)\quad PDWL_X = \{[½(0.2)(-0.020049)(-0.040098)] \times \$1{,}000\}$$
$$+ \{[½(0.5)(-0.000870)(-0.000870)] \times \$1{,}000\}$$
$$+ \{[½(0.3)(0.012403)(0.002481)] \times \$1{,}000\}$$
$$= (0.0000804 \times \$1{,}000) + (0.0000002 \times \$1{,}000)$$
$$+ (0.0000046 \times \$1{,}000)$$
$$= \$0.0804 + \$0.0002 + \$0.0046 = \$0.085.$$

Total Changes in Surplus Caused by a Tax on an Input

The change in consumer surplus as indicated by (8.91) is –\$19.20. The total change in producer surplus given by (8.95) is –\$0.64, and the change in total surplus (ΔTS) is

$$(8.100)\quad \Delta TS = \Delta CS + \Delta PS_X = -\$19.20 - \$0.64 = -\$19.84.$$

These values were calculated by considering changes in consumer surplus at the output level and summing the changes in producer surplus across the three input markets. This approach allows for calculating the incidence of changes in surplus across all market participants. However, it is possible to approximate changes in net total surplus using only the input market in which the tax was imposed. To illustrate, we use the concept of derived demand surplus as noted in (8.70), such that

$$(8.101)\quad \Delta DDS_{x_1} = [-(0.2)(0.079951)\{1 + ½(-0.040098)\}] \times \$1{,}000$$
$$= (-0.015670 \times \$1{,}000) = -\$15.67.$$

Equation (8.71) indicates that ΔDDS_{x_1} is also approximately equal to

$$(8.102)\quad \Delta DDS_{x_1} \approx \Delta CS + \textstyle\sum_{i=2}^{3} \Delta PS_{x_i}$$
$$= -\$19.20 - \$0.43 + \$3.73 = -\$15.91.$$

Note that the values in (8.101) and (8.102) are not identical, in that

(8.103) $\Delta DDS_{x_1} - \Delta CS + \sum_{i=2}^{3} \Delta PS_{x_i} = \$0.24.$

The difference between the two measures is represented by (8.80), such that

(8.104) $$½\boldsymbol{KEW}'\boldsymbol{\Omega KEW} =$$

$$½[0.015990 \ \ -0.000435 \ \ 0.003721]\begin{bmatrix} -2.25 & 0.30 & 1.00 \\ 0.30 & -0.42 & 0.50 \\ 1.00 & 0.50 & -1.50 \end{bmatrix}\begin{bmatrix} 0.015990 \\ -0.000435 \\ 0.003721 \end{bmatrix}$$

$$= \$0.24.$$

Thus, subject to this small error, the derived demand surplus calculated in market for which the tax was imposed is approximately equal to the net changes in producer and consumer surpluses in the nontaxed factor and output markets. Hence, the total net change in surplus for the market is calculated using (8.101) and (8.95) as

(8.105) $\Delta TS \approx \Delta DDS_{x_1} + \Delta PS_{x_1} = -\$15.67 - \$3.93 = -\$19.60.$

Note that this is very close to the value calculated in (8.100), with the difference being equal to that indicated in (8.104).

Total Net Tax Receipts from a Tax on an Input

Total net tax receipts are calculated by summing those paid by consumers and those paid by input suppliers. Input market 3 is a recipient of an implicit tax subsidy caused by the excise tax imposed on input 1. Total tax receipts (*TTAX*) are calculated using (8.92) and (8.97):

(8.106) $TTAX = CTAX + PTAX_X = \$19.13 + \$0.55 = \$19.68.$

Total net tax receipts can be approximated by considering only the input market for which the tax was imposed. However, this approach cannot be used to calculate the incidence of the tax burden among the input markets. Using only the input market in which the tax was imposed, derived demand tax (*DDTAX*) receipts are calculated as

(8.107) $$DDTAX_{x_1} = [(0.2)(0.079951)\{1 + (-0.040098)\}] \times \$1{,}000$$

$$= 0.015349 \times \$1{,}000 = \$15.35.$$

The value indicated in (8.107) is approximately equal to the net taxes paid and received by consumers and the producers of inputs 2 and 3 in (8.83), which results in

$$(8.108)\quad DDTAX_{x_1} \approx CTAX + \sum_{i=2}^{3} PTAX_{x_i}$$

$$= \$19.13 + \$0.43 - \$3.73 = \$15.83.$$

The values calculated in (8.107) and (8.108) are not identical, and the \$0.48 difference is equal to

$$(8.109)\quad DDTAX_{x_1} - \left(CTAX + \sum_{i=2}^{n} PTAX_{x_i}\right) \approx \boldsymbol{KEW'\Omega KEW} =$$

$$[0.015990 \quad -0.000435 \quad 0.003721]\begin{bmatrix} -2.25 & 0.30 & 1.00 \\ 0.30 & -0.42 & 0.50 \\ 1.00 & 0.50 & -1.50 \end{bmatrix}\begin{bmatrix} 0.015990 \\ -0.000435 \\ 0.001372 \end{bmatrix}$$

$$= \$0.48$$

Subject to this small error, net tax receipts can be calculated by considering only the input market to which the tax was applied.

Total Net Deadweight Losses Caused by a Tax on an Input

Total net deadweight losses (*DWL*) are calculated by summing those incurred by consumers and all input suppliers using (8.93) and (8.99):

$$(8.110)\quad DWL = CDWL + PDWL_X = \$0.074 + \$0.085 = \$0.16.$$

Total net deadweight losses can also be approximated by considering only the input market for which the tax was imposed. However, this approach cannot be used to calculate the incidence of deadweight losses. Equation (8.86) shows that the calculation is given by

$$(8.111)\quad DDDWL_{x_1} = [-½K_1 E(w_1^D)E(x_1)]p_0 q_0$$

$$= [-½(0.20)(0.079951)(-0.040098)] \times \$1{,}000$$

$$= 0.000321 \times \$1{,}000 = \$0.32.$$

The value indicated in (8.111) is approximately equal to the net deadweight loss (or gain) incurred by consumers and the producers of inputs 2 and 3 using (8.87):

(8.112) $DDDWL_{x_1} \approx CDWL + \sum_{i=2}^{3} PDWL_{x_i}$

$$= \$0.0743 + \$0.0002 + \$0.0046 = \$0.08.$$

Note that the values in (8.111) and (8.112) differ by \$0.24, which is explained by (8.88):

(8.113) $DDDWL_{x_1} - \left(CDWL + \sum_{i=2}^{3} PDWL_{x_i}\right) \approx -½\, \boldsymbol{KEW'\Omega KEW} =$

$$½[0.015990 \;\; -0.000435 \;\; 0.003721]\begin{bmatrix} -2.25 & 0.30 & 1.00 \\ 0.30 & -0.42 & 0.50 \\ 1.00 & 0.50 & -1.50 \end{bmatrix}\begin{bmatrix} 0.015990 \\ -0.000435 \\ 0.003721 \end{bmatrix}$$

$= \$0.24.$

Thus, the net change in deadweight losses calculated in the factor market in which the tax was imposed is given by

(8.114) $DWL \approx DDDWL_{x_1} + PDWL_{x_1} = \$0.32 + \$0.08 = \$0.40.$

A Two-Output, Three-Input Model

We expand the previous EDMs by considering an economic system that includes the production of two wholesale commodities such as beef and pork. We assume that both wholesale products are produced using a raw agricultural commodity—cattle in the case of beef and hogs in the case of pork. In addition, we assume that labor is required to produce wholesale beef and pork products, that labor is not specialized in terms of beef or pork production, and that substitution possibilities exist between labor and the two agricultural commodities. In addition, cattle cannot be substituted for hogs in the production of pork and hogs cannot be substituted for cattle in the production of beef.

For the beef sector, the wholesale demand and supply functions are given by

(8.115) $q_b^D = q_b^D(p_b^D)$ wholesale beef derived demand

(8.116) $q_b^S = f(x_c^D, x_{lb}^D)$ wholesale beef production

where q_b^D is the quantity demanded of wholesale beef, p_b^D is the wholesale price of beef, q_b^S is the quantity supplied of beef, x_c^D is the quantity demanded of cattle used to produce beef, and x_{lb}^D is the quantity demanded of labor used to process cattle into beef products. The first-order conditions (FOCs) for beef production are given by

(8.117) $p_b^S f_c - w_c^D = 0$

(8.118) $p_b^S f_l - w_l^D = 0,$

where p_b^S is the supply price of beef, f_c is the partial derivative of the production function (8.116) with respect to x_c^D, w_c^D is the demand price of cattle inputs, f_l is the partial derivative of the production function (8.116) with respect to x_{lb}^D, and w_l^D is the demand price of labor.

For the pork sector, the general demand and supply functions are given by

(8.119) $q_p^D = q_p^D(p_p^D)$ wholesale pork derived demand

(8.120) $q_p^S = g(x_h^D, x_{lp}^D),$ wholesale pork production

where q_p^D is the quantity demanded of pork, p_p^D is the wholesale price of pork, q_p^S is the quantity supplied of pork, x_h^D is the quantity demanded of hogs used to produce pork, and x_{lp}^D is the quantity demanded of labor used to process hogs into pork products. The FOCs for pork production are given by

(8.121) $p_p^S g_h - w_h^D = 0$

(8.122) $p_p^S g_l - w_l^D = 0,$

where p_p^S is the supply price of pork, g_h is the partial derivative of the production function (8.120) with respect to x_h^D, w_h^D is the demand price of hog inputs, g_l is the partial derivative of the production function (8.120) with respect to x_{lp}^D, and w_l^D is the demand price of labor.

The total amount of labor used by the two sectors is the sum of the amount used to produce beef and the amount used to produce pork:

(8.123) $x_l^D = x_{lb}^D + x_{lp}^D.$

The supplies of cattle, x_c^S, hogs, x_h^S, and labor, x_l^S, are assumed to be functions of only their respective prices, w_c^S, w_h^S, and w_l^S:

(8.124) $x_c^S = x_c^S(w_c^S)$ supply of cattle

(8.125) $x_h^S = x_h^S(w_h^S)$ supply of hogs

(8.126) $x_l^S = x_l^S(w_l^S).$ supply of labor

The EDM for this problem is written as

(8.127) $E(q_b^D) = \eta^B E(p_b^D) + E(\theta_1)$

(8.128) $E\left(q_p^D\right) = \eta^P E(p_b^D) + E(\theta_2)$

(8.129) $E(p_b^S) = K_c^b E(w_c^D) + K_l^b E(w_l^D) + E(\theta_3)$

(8.130) $E\left(p_p^S\right) = K_h^p E(w_h^D) + K_l^p E(w_l^D) + E(\theta_4)$

(8.131) $E(x_c^D) = E(q_b^S) + K_c^b \sigma_{cc} E(w_c^D) + K_l^b \sigma_{cl} E(w_l^D) + E(\theta_5)$

(8.132) $E(x_{lb}^D) = E(q_b^S) + K_c^b \sigma_{lc} E(w_c^D) + K_l^b \sigma_{ll}^b E(w_l^D) + E(\theta_6)$

(8.133) $E(x_h^D) = E\left(q_p^S\right) + K_h^p \sigma_{hh} E(w_h^D) + K_l^p \sigma_{hl} E(w_l^D) + E(\theta_7)$

(8.134) $E\left(x_{lp}^D\right) = E\left(q_p^S\right) + K_h^p \sigma_{lh} E(w_h^D) + K_l^p \sigma_{ll}^p E(w_l^D) + E(\theta_8)$

(8.135) $E(x_l^D) = \left(\frac{x_{lb}^D}{x_l^D}\right) E(x_{lb}^D) + \left(\frac{x_{lp}^D}{x_l^D}\right) E\left(x_{lp}^D\right)$

(8.136) $E(x_c^S) = \varepsilon_c E(w_c^S) + E(\theta_9)$

(8.137) $E(x_h^S) = \varepsilon_h E(w_h^S) + E(\theta_{10})$

(8.138) $E(x_l^S) = \varepsilon_l E(w_l^S) + E(\theta_{11}),$

where η^B is the own-price elasticity of demand for wholesale beef, η^P is the own-price elasticity of demand for wholesale pork, K_c^b is the factor share of cattle used to produce beef, K_l^b is the factor share of labor used to produce beef, K_h^p is the factor share of hogs used to produce pork, K_l^p is the factor share of labor used to produce pork, σ_{cl} is the AES between cattle and labor used to produce beef (which is equal to σ_{lc}), and σ_{hl} is the AES between hogs and labor used to produce pork (which is equal to σ_{lh}). The term x_{lb}^D / x_l^D represents the proportion of labor used by the beef production sector and x_{lp}^D / x_l^D represents the proportion of labor used by the pork production sector.

The following equilibrium equations are added to the EDM so policy actions that cause wedges to occur between output and input demand and supply quantities and prices can be considered:

(8.139) $E(q_b^D) = E(q_b^s) + E(\theta_{12})$

(8.140) $E(q_p^D) = E(q_p^S) + E(\theta_{13})$

(8.141) $E(p_b^D) = E(p_b^S) + E(\theta_{14})$

(8.142) $E(p_p^D) = E(p_p^S) + E(\theta_{15})$

(8.143) $E(x_c^D) = E(x_c^S) + E(\theta_{16})$

(8.144) $E(x_h^D) = E(x_h^S) + E(\theta_{17})$

(8.145) $E(x_l^D) = E(x_l^S) + E(\theta_{18})$

(8.146) $E(w_c^D) = E(w_c^S) + E(\theta_{19})$

(8.147) $E(w_h^D) = E(w_h^S) + E(\theta_{20})$

(8.148) $E(w_l^D) = E(w_l^S) + E(\theta_{21})$.

Moving the endogenous variables in (8.127)–(8.148) to the left-hand side results in

(8.149) $E(q_b^D) - \eta^B E(p_b^D) = E(\theta_1)$

(8.150) $E(q_p^D) - \eta^P E(p_b^D) = E(\theta_2)$

(8.151) $E(p_b^S) - K_c^b E(w_c^D) - K_l^b E(w_l^D) = E(\theta_3)$

(8.152) $E(p_p^S) - K_h^p E(w_h^D) - K_l^p E(w_l^D) = E(\theta_4)$

(8.153) $E(x_c^D) - E(q_b^S) - K_c^b \sigma_{cc} E(w_c^D) - K_l^b \sigma_{cl} E(w_l^D) = E(\theta_5)$

(8.154) $E(x_{lb}^D) - E(q_b^S) - K_c^b \sigma_{lc} E(w_c^D) - K_l^b \sigma_{ll}^b E(w_l^D) = E(\theta_6)$

(8.155) $E(x_h^D) - E(q_p^S) - K_h^p \sigma_{hh} E(w_h^D) - K_l^p \sigma_{hl} E(w_l^D) = E(\theta_7)$

(8.156) $E(x_{lp}^D) - E(q_p^S) - K_h^p \sigma_{lh} E(w_h^D) - K_l^p \sigma_{ll}^p E(w_l^D) = E(\theta_8)$

(8.157) $E(x_l^D) - \left(\frac{x_{lb}^D}{x_l^D}\right) E(x_{lb}^D) - \left(\frac{x_{lp}^D}{x_l^D}\right) E(x_{lp}^D) = 0$

(8.158) $E(x_c^S) - \varepsilon_c E(w_c^S) = E(\theta_9)$

(8.159) $E(x_h^S) - \varepsilon_h E(w_h^S) = E(\theta_{10})$

(8.160) $E(x_l^S) - \varepsilon_l E(w_l^S) = E(\theta_{11})$

(8.161) $E(q_b^D) - E(q_b^S) = E(\theta_{12})$

(8.162) $E(q_p^D) - E(q_p^S) = E(\theta_{13})$

(8.163) $E(p_b^D) - E(p_b^S) = E(\theta_{14})$

(8.164) $E(p_p^D) - E(p_p^S) = E(\theta_{15})$

(8.165) $E(x_c^D) - E(x_c^S) = E(\theta_{16})$

(8.166) $E(x_h^D) - E(x_h^S) = E(\theta_{17})$

(8.167) $E(x_l^D) - E(x_l^S) = E(\theta_{18})$

(8.168) $E(w_c^D) - E(w_c^S) = E(\theta_{19})$

(8.169) $E(w_h^D) - E(w_h^S) = E(\theta_{20})$

(8.170) $E(w_l^D) - E(w_l^S) = E(\theta_{21})$.

Putting (8.149)–(8.170) into general matrix notation results in

(8.171) $\boldsymbol{Ay} = \boldsymbol{b}$,

where $\boldsymbol{y}$ is a 22×1 vector of endogenous variables, $\boldsymbol{A}$ is a 22×22 matrix of parameters, and $\boldsymbol{b}$ is a 22×1 vector of exogenous shocks. The $\boldsymbol{y}$ and $\boldsymbol{b}$ vectors are given by

$$(8.172)\quad \boldsymbol{y} = \begin{bmatrix} E(q_b^D) \\ E(q_b^S) \\ E(q_p^D) \\ E(q_p^S) \\ E(p_b^D) \\ E(p_p^S) \\ E(p_p^D) \\ E(p_p^S) \\ E(x_c^D) \\ E(x_{lb}^D) \\ E(x_h^D) \\ E(x_{lp}^D) \\ E(x_l^D) \\ E(x_c^S) \\ E(x_h^S) \\ E(x_l^S) \\ E(w_c^D) \\ E(w_h^D) \\ E(w_l^D) \\ E(w_c^S) \\ E(w_h^S) \\ E(w_l^S) \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} E(\theta_1) \\ E(\theta_2) \\ E(\theta_3) \\ E(\theta_4) \\ E(\theta_5) \\ E(\theta_6) \\ E(\theta_7) \\ E(\theta_8) \\ 0 \\ E(\theta_9) \\ E(\theta_{10}) \\ E(\theta_{11}) \\ E(\theta_{12}) \\ E(\theta_{13}) \\ E(\theta_{14}) \\ E(\theta_{15}) \\ E(\theta_{16}) \\ E(\theta_{17}) \\ E(\theta_{18}) \\ E(\theta_{19}) \\ E(\theta_{20}) \\ E(\theta_{21}) \end{bmatrix}.$$

To populate the ***A*** matrix, we assume that the own-price elasticity of demand for beef, η^B, is −0.60 and the own-price elasticity of demand for pork, η^P, is −0.40. In addition, we assume the own-price elasticity of supply of cattle, ε_c, is 0.40; the own-price elasticity of supply of hogs, ε_h, is 0.60; and the own-price elasticity of supply of labor, ε_l, is 0.80.

We also assume that the factor share of cattle, K_c^b, used to produce beef is 0.90 and the factor share of hogs, K_h^p, used to produce pork is 0.80. Thus, the factor share of labor used to produce beef, K_l^b, is 0.10, and the factor share of labor used to produce pork, K_l^p, is 0.20. The AES between cattle and labor, σ_{cl}, used in the production of beef is assumed to equal 0.40, so that $\sigma_{cl} = \sigma_{lc} = 0.40$. While U.S. beef and pork production quantities are relatively similar on an annual basis, almost 4 times more hogs are slaughtered relative to cattle. Thus, we assume that pork production is relatively more labor intensive than beef production. The AES between hogs and labor, σ_{hl}, in the production of pork is assumed to equal 0.30, so that $\sigma_{hl} = \sigma_{lh} = 0.30$, which is smaller than the AES between cattle and labor.

The remaining AES values are calculated for the beef production sector as

$$\sigma_{cc} = -\frac{K_l^b \sigma_{cl}}{K_c^b} = -0.044 \text{ and } \sigma_{ll}^b = -\frac{K_c^b \sigma_{lc}}{K_l^b} = -3.60.$$

For the pork production sector, they are calculated as

$$\sigma_{hh} = -\frac{K_l^p \sigma_{hl}}{K_h^p} = -0.075 \text{ and } \sigma_{ll}^p = -\frac{K_h^p \sigma_{lh}}{K_l^p} = -1.20.$$

We use average annual data for the period 2015–2019 to represent the sizes of the industries being modeled. Over this time period, the total value of cattle production averaged \$52,200 million annually, while the total value of hog production averaged about \$18,600 million. Given the perfect competition assumption of zero economic profits and factor share values, K_c^b indicates that the value of cattle represents 90% of the wholesale value of beef. Thus, the estimated size of the wholesale beef industry is given by

(8.173) $p_0^b q_0^b = \$52{,}200 \text{ million} / 0.90 = \$58{,}000 \text{ million}.$

Likewise, K_h^p indicates the size of the wholesale pork industry is given by

(8.174) $p_0^p q_0^p = \$18{,}600 \text{ million} / 0.80 = \$23{,}250 \text{ million}.$

Therefore, the size of the labor sector used to produce wholesale beef is given by

(8.175) $w_0^l x_0^{lb} = K_l^b \left(p_0^b q_0^b\right) = 0.10 \times \$58{,}000 \text{ million} = \$5{,}800 \text{ million}.$

The size of the labor sector used to produce wholesale pork is given by

(8.176) $w_0^l x_0^{lp} = K_l^p \left(p_0^p q_0^p\right) = 0.20 \times \$23{,}250 \text{ million} = \$4{,}600 \text{ million}.$

Thus, total labor expenditures used to produce wholesale beef and pork equals \$10,450 million (\$5,800 million + \$4,650 million). The beef share of total labor expenditures is given by

(8.177) $\frac{x_{lb}^D}{x_l^D} = \frac{\$5{,}800}{\$10{,}450} = 0.56$

and the pork share of total labor expenditures is given by

$$\frac{x_{lp}^D}{x_l^D} = \frac{\$4{,}650}{\$10{,}450} = 0.44. \quad (8.178)$$

Using these values, the $\boldsymbol{A}$ matrix is populated as

(8.179)

1	0	0	0	0.60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0.40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	-0.90	0	-0.10	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-0.80	-0.20	0	0	0
0	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0.04	0	-0.04	0	0	0
0	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-0.36	0	0.36	0	0	0
0	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	-0.06	0.06	0	0	0
0	0	0	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0.24	-0.24	0	0	0
0	0	0	0	0	0	0	0	0	-0.56	0	-0.44	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-0.40	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-0.60	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-0.80
1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1

Homogeneity of the Production Processes

One way to check for the theoretical and empirical consistency of an EDM is to ascertain whether the system meets the homogeneity of degree 0 (HD0) requirement. The process involves placing a price wedge of, say 10%, between each input supply and demand price in the form of a tax. In addition, the same price wedge is simultaneously placed between each of the output demand and output supply prices in the form of a producer subsidy. The price wedges placed in the input markets are entered as positive values in the $\boldsymbol{b}$ vector to represent an increase in the demand price of the inputs caused by the tax. However, the price wedges placed in the output markets are entered as negative values in the $\boldsymbol{b}$ vector to represent an increase in the price that producers receive for their output because of the subsidy. If the model is HD0 in input and output prices, then identical increases in output and input prices will not have any impact on equilibrium output or input quantities. Thus, the test is conducted using the following $\boldsymbol{b}$ vector:

$$(8.180)\qquad \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.10 \\ -0.10 \\ 0 \\ 0 \\ 0 \\ 0.10 \\ 0.10 \\ 0.10 \end{bmatrix}.$$

Solving (8.171) as

$$(8.181)\qquad \boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{b}$$

results in changes in endogenous variables presented in (8.182). A 10% increase in the demand price of the three inputs (w_c^D, w_h^D, w_l^D) coupled with a 10% increase in the price of beef and pork products received by producers (p_b^S, p_p^S) does not cause any other changes in the endogenous variables of the economic system. Therefore, the homogeneity condition is met.

$$\mathbf{y} = \begin{bmatrix} E(q_b^D) \\ E(q_b^S) \\ E(q_p^D) \\ E(q_p^S) \\ E(p_b^D) \\ E(p_b^S) \\ E(p_p^D) \\ E(p_p^S) \\ E(x_c^D) \\ E(x_{lb}^D) \\ E(x_h^D) \\ E(x_{lp}^D) \\ E(x_l^D) \\ E(x_c^S) \\ E(x_h^S) \\ E(x_l^S) \\ E(w_c^D) \\ E(w_h^D) \\ E(w_l^D) \\ E(w_c^S) \\ E(w_h^S) \\ E(w_l^S) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.10 \\ 0 \\ 0.10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{8.182}$$

A 10% Tax on Labor

Suppose a payroll tax of 10% is placed on labor inputs (input 3). The exogenous shock places a price wedge between the demand and supply prices of labor and is implemented by setting $E(\theta_{21})$ equal to 0.10 in the $\boldsymbol{b}$ vector and all other entries equal to 0. The effect on the endogenous variables is given by

$$(8.183)\quad \boldsymbol{y} = \begin{bmatrix} E(q_b^D) \\ E(q_b^S) \\ E(q_p^D) \\ E(q_p^S) \\ E(p_b^D) \\ E(p_p^S) \\ E(p_p^D) \\ E(p_p^S) \\ E(x_c^D) \\ E(x_{lb}^D) \\ E(x_h^D) \\ E(x_{lp}^D) \\ E(x_l^D) \\ E(x_c^S) \\ E(x_h^S) \\ E(x_l^S) \\ E(w_c^D) \\ E(w_h^D) \\ E(w_l^D) \\ E(w_c^S) \\ E(w_h^S) \\ E(w_l^S) \end{bmatrix} = \begin{bmatrix} -0.003341 \\ -0.003341 \\ -0.005012 \\ -0.005012 \\ 0.005569 \\ 0.005569 \\ 0.012530 \\ 0.012530 \\ -0.000557 \\ -0.028400 \\ -0.000835 \\ -0.021718 \\ -0.025427 \\ -0.000557 \\ -0.000835 \\ -0.025427 \\ -0.001392 \\ -0.001392 \\ 0.068216 \\ -0.001392 \\ -0.001392 \\ -0.031784 \end{bmatrix}.$$

The 10% tax on labor causes its supply price to decline by 3.2% and its demand price to increase by 6.8%. The sum of the absolute value of these two effects represents the 10% tax wedge that was placed between the demand and supply prices. Note that the purchasers of labor bear a larger incidence of the tax than the suppliers of labor.

Because the price of labor increases, the beef and pork processing sectors use less labor. The total reduction of labor use, $E(x_l^D) = E(x_l^S)$, is 2.5%. The amount of labor used in beef production, $E(x_{lb}^D)$, declines by 2.8% and the amount used in pork production, $E(x_{lp}^D)$, declines by 2.2%. Although the pork production sector is more labor intensive per unit of output, the larger scale of the beef sector results in its use of 56% of all labor inputs while the pork production sector uses 44%. Thus, the share-weighted averages of the two percentage reductions in each sector are equal to the overall percentage reduction in the use of labor.

Because increased labor costs cause the pork industry to reduce its output, the use of hogs, $E(x_h^D)$, decreases by 0.1% and the price of hogs, $E(w_h^D)$, decreases by 0.1%. The tax on labor reduces the production of pork and cause the equilibrium price of pork, $E(p_p^D)$ and $E(p_p^S)$, to increase by 1.3% while the quantity of pork consumed, $E(q_p^D)$, declines by 0.5%. The production of beef, $E(q_b^S)$, decreases by 0.3% while the equilibrium price of beef, $E(p_b^D)$, increases by 0.6%. The reduction in beef production causes 0.1% fewer cattle, $E(x_c^D)$, to be used which reduces the price of cattle, $E(w_c^D)$, by 0.1%.

General Issues Related to Surplus, Tax, and Deadweight Loss Effects of a 10% Tax on Labor

The equilibria trajectories for each market and labor tax wedge are presented in Figure 8.9. We have shown that the surplus effects of the tax can be estimated in two ways. The first approach calculates aggregate surplus changes using only the market in which the tax wedge was imposed, as presented in the lower right panel in Figure 8.9. The second approach uses the output market and each of the non-taxed input markets to calculate changes in surplus. This latter approach disaggregates the total effects so that the incidence of the effects in each market can be calculated. As previously demonstrated, the two approaches provide almost identical aggregate results.

In previous examples using one output, surplus results were presented as proportions of the initial size or total revenue of the industry. Although other scaling

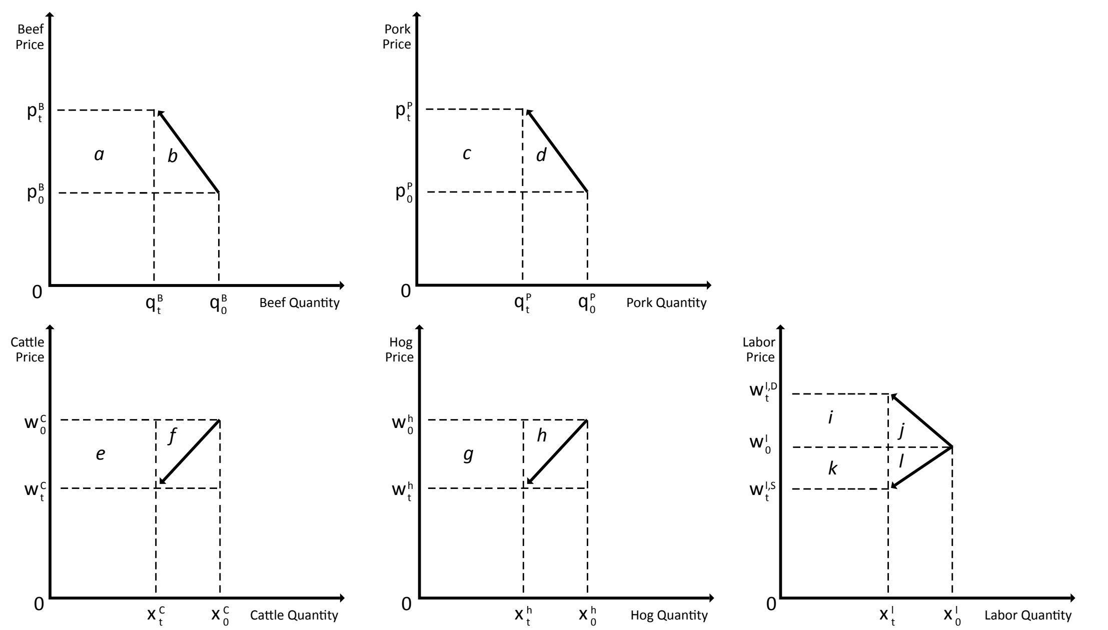

Figure 8.9. Two-Output Model with a Tax Placed on a Factor Input

metrics can be used, the purpose of using industry size was to facilitate the derivation of errors between the two approaches in terms of quadratic forms. Because of the addition of a second output market, we use the size of the output and input markets to calculate proportional changes in surplus.

Changes in Surplus Caused by a 10% Tax on Labor

We use the market in which the tax was placed to calculate changes in surplus using the second expression in (8.70). The change in derived demand surplus is given by

$$(8.184)\quad \Delta DDS_l = -\text{area}\ (i+j) = [-E(w_l^D)½\{1+E(x_l^D)\}]w_0^l x_0^l$$
$$= [-(0.068216)\{1+½(-0.025427)\}] \times \$10{,}450$$
$$= -0.067349 \times \$10{,}450 = -\$703.80 \text{ million.}$$

Using the second expression in (8.59), the change in producer surplus in the labor market is given by

$$(8.185)\quad \Delta PS_l = -\text{area}\ (k+l) = [E(w_l^S)\{1+½E(x_l^S)\}]w_0^l x_0^l$$
$$= [-0.031789\{1+½(-0.025427)\}] \times \$10{,}450$$
$$= (-0.031379) \times \$10{,}450 = -\$327.92 \text{ million.}$$

The sum of (8.184) and (8.185) represents the aggregate change in total surplus for consumers and all producers, a reduction of $1,031.72 million, that results from the imposition of a 10% tax in the labor market. Note that other than for the loss of producer surplus in the labor market, no other single market surplus change is identified when calculating changes using only the market that was taxed. To illustrate how the change in derived demand surplus is approximately equal to the sum of the consumer and producer surplus changes in the nonlabor markets, consider

$$(8.186)\quad \Delta DDS_l \approx -\text{area}\ (i+j) \approx -\text{area}\ (a+b) - \text{area}\ (c+d)$$
$$- \text{area}\ (e+f) - \text{area}\ (g+h)$$

or

$$(8.187)\quad \Delta DDS_l \approx \Delta CS_b + \Delta CS_p + \Delta PS_c + \Delta PS_h.$$

The change in beef consumer surplus is calculated using (8.56)

$$\text{(8.188)}\quad \Delta CS_b = \left[-\left(E(p_b^D)\right)\{1 + ½E(q_b^D)\}\right] p_0^b q_0^b$$

$$= [(-0.005569)\{1 + ½(-0.003341\}] \times \$58{,}000$$

$$= (-0.005559) \times \$58{,}000 = -\$322.44 \text{ million.}$$

For the pork market, the change in consumer surplus is given by:

$$\text{(8.189)}\quad \Delta CS_p = \left[-\left(E\left(p_p^D\right)\right)\{1 + ½E\left(q_p^D\right)\}\right] p_0^p q_0^p$$

$$= [(-0.012530)\{1 + ½(-0.005569\}] \times \$23{,}250$$

$$= (-0.012498) \times \$23{,}250 = -\$290.58 \text{ million.}$$

The change in producer surplus in the cattle market, ΔPS_c, is calculated using the second expression in (8.59) as

$$\text{(8.190)}\quad \Delta PS_c = [E(w_c^S)\{1 + ½E(x_c^S)\}] w_0^c x_0^c$$

$$= [-0.001392\{1 + ½(-0.000557)\}] \times \$52{,}200 \text{ million.}$$

$$= (-0.001391) \times \$52{,}200 = -\$72.65 \text{ million.}$$

Likewise, the change in producer surplus in the hog market, ΔPS_h, is calculated as:

$$\text{(8.191)}\quad \Delta PS_h = [E(w_h^S)\{1 + ½E(x_h^S)\}] w_0^h x_0^h$$

$$= [-0.001392\{1 + ½(-0.000835)\}] \times \$18{,}600 \text{ million.}$$

$$= (-0.001391) \times \$18{,}600 = -\$25.88 \text{ million.}$$

Substituting the results of (8.188), (8.189), (8.190), and (8.191) into (8.187) yields

$$\text{(8.192)}\quad \Delta DDS_l \approx -\$322.44 - \$290.58 - \$72.66 - \$25.88$$

$$= -\$711.56 \text{ million.}$$

Note that the difference in ΔDDS_l as calculated in (8.184) and the value calculated in (8.192) is only \$7.76 million, which is approximately 0.0096% of the total size of beef and pork markets (\$81,250 million). However, this second approach to calculating the total change in surplus allows one to calculate the incidence of surplus in each input market.

Tax Receipts Resulting from a 10% Tax on Labor

Using (8.85), total tax receipts, but not the incidence of taxes, can be calculated by considering only the market in which the tax was imposed:

(8.193) $$TTAX = \text{area } (i + k) = DDTAX_l + PTAX_l.$$

Using the second expression in (8.82), the first term in (8.193) is calculated as:

(8.194) $$\begin{aligned} DDTAX_l = \text{area } (i) &= [E(w_l^D)\{1 + E(x_l^D)\}]w_0^l x_0^l \\ &= [0.068216\{1 + (-0.025427)\}] \\ &\quad \times \$10{,}450 \text{ million.} \\ &= (-0.066482) \times \$10{,}450 \\ &= \$694.74 \text{ million.} \end{aligned}$$

The second term in (8.193) is obtained using the second expression in (8.66) as:

(8.195) $$\begin{aligned} PTAX_l = \text{area } (k) &= [-E(w_l^S)\{1 + E(x_l^S)\}]w_0^l x_0^l \\ &= [-(-0.031784)\{1 + (-0.025427)\}] \\ &\quad \times \$10{,}450 \text{ million.} \\ &= (0.030975) \times \$10{,}450 \\ &= \$323.69 \text{ million.} \end{aligned}$$

Hence, total tax receipts generated by the 10% tax in the labor market can be calculated as

(8.196) $$\begin{aligned} TTAX &= DDTAX_l + PTAX_l = \$694.74 + \$323.69 \\ &= \$1{,}018.43 \text{ million.} \end{aligned}$$

The amount of the tax paid by beef consumers is calculated using (8.57) as:

$$(8.197) \quad CTAX_b = \text{area } (a) = \left[\left(E(p_b^D)\right)\{1 + E(q_b^D)\}\right] p_0^b q_0^b$$

$$= [(0.005569)\{1 + (-0.003341)\}]$$

$$\times \$58{,}000$$

$$= (0.005550) \times \$58{,}000$$

$$= \$321.91 \text{ million.}$$

Likewise, the amount of the tax paid by pork consumers is given by

$$(8.198) \quad CTAX_p = \text{area } (c) = \left[\left(E(p_p^D)\right)\{1 + E\left(q_p^D\right)\}\right] p_0^p q_0^p$$

$$= [(0.012530)\{1 + (-0.005012)\}]$$

$$\times \$23{,}250 \text{ million.}$$

$$= (0.012467) \times \$23{,}250$$

$$= \$289.85 \text{ million.}$$

The amount of taxes paid by the cattle sector is obtained using the second expression in (8.66):

$$(8.199) \quad PTAX_c = \text{area } (e) = [-E(w_c^S)\{1 + E(x_c^S)\}] w_0^c x_0^c$$

$$= [-(-0.001392)\{1 - 0.000557)\}]$$

$$\times \$52{,}200$$

$$= (0.00139) \times \$52{,}200$$

$$= \$72.63 \text{ million.}$$

while the hog sector's tax payments are similarly calculated as

(8.200) $$PTAX_h = \text{area }(g) = [-E(w_h^S)\{1 + E(x_h^S)\}]w_0^h x_0^h$$
$$= [-(-0.001392)\{-0.000835)\}]$$
$$\times \$18{,}600$$
$$= (0.001391) \times \$18{,}600$$
$$= \$25.87 \text{ million.}$$

Summing the results of (8.197), (8.198), (8.199), and (8.200) results in

(8.201) $$DDTAX_l = CTAX_b + CTAX_p + PTAX_c + PTAX_h$$
$$= \$710.26 \text{ million.}$$

Note that subtracting (8.201) from (8.194) results in a difference of \$15.52 million, which shows that the difference between the two approaches to calculating tax receipts is relatively small and amounts to about 1.5% of total estimated taxes. If a researcher is only interested in estimating total tax receipts, then using (8.193), (8.194), and (8.195) is an appropriate approach. However, if the tax incidence on each sector is desired, (8.199), (8.200), and (8.201) should be employed.

Deadweight Losses Resulting from a 10% Tax on Labor

The aggregate deadweight losses caused by the 10% tax on labor can be calculated using only the market in which the tax was imposed. In this case, total deadweight losses are calculated using (8.89):

(8.202) $$DWL = \text{area }(j + l) = DDDWL_l + PDWL_l.$$

Using the second expression in (8.86), the derived demand deadweight losses can be calculated as:

(8.203) $$DDDWL_l = \text{area } (j) = [-½E(w_l^D)E(x_l^D)]w_0^l x_0^l$$

$$= [-½(0.068216)(-0.025427)]$$

$$\times \$10{,}450 \text{ million.}$$

$$= (0.000867) \times \$10{,}450$$

$$= \$9.06 \text{ million.}$$

The labor market deadweight losses are calculated using the second expression in (8.65)

(8.204) $$PDWL_l = \text{area } (l) = [½E(w_l^S)E(x_l^S)]w_0^l x_0^l$$

$$= [½(-0.031784)(-0.025427)]$$

$$\times \$10{,}450 \text{ million.}$$

$$= (0.000404) \times \$10{,}450$$

$$= \$4.22 \text{ million.}$$

Thus, total deadweight losses can be calculated in (8.202) as

(8.205) $$DWL = DDDWL_l + PDWL_l$$

$$= \$9.06 + \$4.22 = \$13.29 \text{ million.}$$

The incidence of deadweight losses among the input markets can be calculated using the equations developed in the previous section.

Summary

A common objective of policy analyses is to estimate the effect of legislative actions on consumer and producer surplus, the incidence of costs and benefits, and societal deadweight losses or gains. Estimating these effects is complicated because a policy may affect several horizontally or vertically linked market sectors. EDMs are useful tools for assessing these effects on systems of linked markets. This chapter demonstrates that EDMs linearly approximate sector-specific price-quantity equilibria along total-response trajectories. The equilibria changes can be used to estimate the levels and incidence of changes in surplus, tax receipts, and deadweight losses. When a market-specific policy is implemented, "wedges" are often driven between equilibrium prices or quantities in that market. In this case, total changes in surplus, taxes, and deadweight losses can be estimated by integrating equilibria trajectories solely within the market that contains the policy-induced "wedge."

Alternatively, the incidence of changes in surplus, tax receipts, and deadweight losses can be estimated by separately integrating the linearly approximated equilibrium trajectories in each market if a tax was imposed on an input. When summed, these individual effects are shown to closely approximate the aggregate metrics computed using only the market in which the price or quantity wedge occurred. Note that if a tax is placed on an input, changes in producer surplus cannot be calculated in the output market because the consumer demand price will continue to equal the producer supply price at that level.

Although EDMs have proven useful for many applications, they have some limitations for estimating surplus effects. Changes in price and quantity equilibria occur along linearly tangent hyperplanes or total differentials in price-quantity space. As such, EDMs as well as underlying total differential linear approximations are most useful for estimating changes caused by relatively small policy or system shocks.

Similar to other partial equilibrium models, researchers considering the effects of market interventions must decide which market sectors are to be treated as exogenous versus endogenous in an EDM. When constructing a partial equilibrium model, the model's exogenous market information such as consumers' or input suppliers' price-quantity total-response trajectories are taken as given. Therefore, an EDM can estimate surplus changes in these markets as well as for those that are fully internal to the model.

EDMs cannot, however, be used to estimate surplus changes for a shock emanating from sectors that are external to the model. While an EDM can be used to estimate the producer surplus effects of an exogenous demand shock, EDMs cannot estimate changes in consumer surplus emanating from a demand shock unless the model contains the sectors that are also indirectly affected. That is, a shock to the demand of a product for which substitutes or complements exist affects producer surplus throughout the model. But if the demands for substitutes or complements are not included in an EDM, then changes in consumer surplus calculated within the model ignore positive or negative surplus effects from outside of the model.

»Chapter Nine

SENSITIVITY ANALYSES OF EQUILIBRIUM DISPLACEMENT MODELS

Equilibrium displacement models (EDMs) are used to calculate changes in endogenous variables caused by exogenous economic or policy shocks for a given set of demand, supply, and input substitution elasticities and factor shares. EDM results can be used in some cases to calculate changes in producer and consumer surplus and the incidence of tax receipts and deadweight losses. These estimates are valuable for policy makers and researchers. However, such calculations represent point estimates of each of these measures and are conditional on the selected parameters used to operationalize the models. In many cases, researchers obtain elasticity estimates from other studies that are themselves contingent upon specific time periods, modeling strategies, and estimation procedures. Factor shares are often obtained from data that may have been gathered with some error or across differing time periods. It is rare for a study to simultaneously estimate a supply and demand system to obtain elasticities and concurrent factor share data for the same time period (e.g., Brester, Marsh, and Atwood, 2009).

All economic models have limitations. To the degree that uncertainty exists in parameter estimates, it is useful to consider the effects of alternative parameter values on EDM results. This can be accomplished by systematically altering each parameter individually to evaluate the sensitivity of EDM estimates to a range of values (Brester and Wohlgenant, 1997). This may be all that is required if a researcher is trying to determine if parameter variations substantially alter EDM results. However, if a researcher wishes to estimate confidence intervals or perform hypothesis tests, then they must consider joint realizations of an EDM's underlying parameters (Davis and Espinoza, 2000).

Brester, Marsh, and Atwood (2009) note that a sensitivity analysis of EDM results should not only consider the variability of parameter estimates, but also their potential dependency using covariances or correlations. It is common to assume a multivariate normal distribution for these parameters and jointly estimate or assume the moments of the multivariate parameters' distributions. It may be that estimates of some joint EDM parameters can be simulated using variances and covariances obtained from the econometric estimation of a system of equations. For example, suppose that a researcher has estimated a system of identified equations and computed a set of elasticity estimates: $\eta_{i,k}^{D} = \eta_{i,k}^{D}(\boldsymbol{\beta}|\boldsymbol{X})$ with $\boldsymbol{\beta}{\sim}MVN(\widehat{\boldsymbol{\beta}}, \boldsymbol{\Sigma_{\beta}})$, where $\eta_{i,k}^{D}$ are own- and cross-elasticities of demand, $\boldsymbol{\beta}$ is a vector of parameter estimates, $\boldsymbol{X}$ is a set of variables, and $\boldsymbol{\Sigma_{\beta}}$ is a matrix containing estimated parameter variances and covariances. The notation $\boldsymbol{\beta}{\sim}MVN(\widehat{\boldsymbol{\beta}}, \boldsymbol{\Sigma_{\beta}})$ indicates that $\boldsymbol{\beta}$ is distributed multivariate normal with mean $\widehat{\boldsymbol{\beta}}$ and covariance matrix $\boldsymbol{\Sigma_{\beta}}$. Simulating a joint set of $\eta_{i,k}^{D}$ could then be accomplished by repeated simulations and calculations of $\eta_{i,k}^{D}(\beta_{i,k}{\sim}MVN(\boldsymbol{\beta}, \boldsymbol{\Sigma_{\beta}}))$ with the resulting $\eta_{i,k}^{D}$ values incorporated into EDM computations.

Although Brester, Marsh, and Atwood (2009) use this procedure for a subset of their simulations, it is usually not possible to estimate a complete set of the required joint EDM parameters from a single system of econometric equations. However, flexible copula procedures can be used to: (1) independently estimate and assume marginal EDM parameter distributions for a set of parameters, (2) independently simulate potential parameter realizations using these distributions, and (3) bind the simulated independent marginals into a set of multivariate parameters. The approach used by Brester, Marsh, and Atwood (2009) examined the sensitivity of EDM estimates of producer and consumer surplus to potential joint variability of EDM parameters. Other researchers have also used this method.

Generating Empirical Distributions of EDM Estimates

In this section, we present a simplified example of procedures that can be used to construct confidence intervals for EDM results. The example uses an assumed joint empirical probability distribution of EDM parameters while imposing minimal prior information with respect to demand and supply elasticities. That is, we assume that own-price demand elasticities are never positive, and that own-price supply elasticities are never negative. The method involves specifying marginal distributions of individual parameter estimates and independently simulating a set of N potential realizations for each parameter.[14] We introduce dependency by binding the independently simulated marginal realizations together using Iman and Conover's (1982) normal-copula procedure to create N sets of correlated or

14 The marginal distributions need not be of the same distributional family, which facilitates the imposition of differing inequality restrictions on simulated model parameters.

dependent parameter values. For each of the N sets of joint parameters, we use an EDM to compute a set of endogenous variable outcomes and estimated changes in consumer and producer surplus. This generates N sets of simulated joint EDM results. These empirical outcomes are used to develop means, confidence intervals, and p-values for price and quantity changes and surplus measures.

An Application to a One-Output, Two-Input Model

Equations (5.1)–(5.8) presented a one-output, two-input EDM that was used to obtain estimates of changes in endogenous variables resulting from a 10% excise tax on output. Equation (5.11) presents the EDM results as:

$$\text{(9.1)} \quad \mathbf{y} = \begin{bmatrix} E(q) \\ E(p^D) \\ E(p^S) \\ E(x_1) \\ E(x_2) \\ E(w_1) \\ E(w_2) \end{bmatrix} = \begin{bmatrix} -0.032 \\ 0.053 \\ -0.047 \\ -0.066 \\ -0.039 \\ -0.013 \\ -0.039 \end{bmatrix}.$$

The point estimates indicate that a 10% excise tax would cause the quantity of output, $E(q)$, to decrease by 3.2%; the demand price, $E(p^D)$, to increase by 5.3%; and the supply price, $E(p^S)$, to decrease by 4.7%. The use of input 1, $E(x_1)$, decreases by 6.6% while its price, $E(w_1)$, decreases by 1.3%. The use of input 2, $E(x_2)$, decreases by 3.9% while its price, $E(w_2)$, decreases by the same amount because of the assumed unitary elasticity of supply for input 2. Using (8.25) and assuming a market size of \$1,000, the excise tax causes consumer surplus to decline by \$51.80. Equation (8.26) indicates that producer surplus declined by \$46.62, and (8.27) and (8.28) are used to calculate a \$19.61 decline in producer surplus in input market 1 and a \$27.09 decline in producer surplus in input market 2.

The changes in endogenous variables and surplus measures were calculated assuming an own-price elasticity of demand, η^D, of −0.60; own-price elasticities of input supplies, ε_1 and ε_2, of 0.20 and 1.0; and an Allen elasticity of substitution between inputs of 1.0. However, each of these four elasticities are point estimates. We wish to: (1) examine the effects of varying the elasticity point estimates upon changes in the EDM-estimated endogenous variables and surplus metrics, (2) estimate confidence intervals for the EDM results, and (3) examine the degree to which potential correlation between the EDM's elasticity parameters affects estimated EDM confidence intervals. In the following section, we present a Monte Carlo simulation approach for obtaining confidence intervals for changes in EDM-developed endogenous variables and surplus calculations.

Simulating Joint Realizations of Changes in Endogenous Variables and Consumer and Producer Surplus

The following discussion presents a simplified example for illustrative purposes.[15] We specify parameter variances for demand and supply elasticities by assuming a coefficient of variation and truncating potential parameter values at 4 standard deviations from their mean values. If 4 standard deviations above or below the mean results in positive or negative values for demand and supply elasticities, respectively, the value is set to 0. Of course, if estimates of the variances of these elasticities were available, one would want to use those in the following procedures.

To model and simulate marginal distributions, we use nonstandardized *Beta* distributions because they can flexibly model various distributional shapes while bounding elasticity parameter realizations. The following procedures can use marginals constructed with Bayesian or other methods that incorporate exogenous information. In addition, the marginals need not be limited to members of the same family.[16]

Let $\boldsymbol{b}$ denote a vector of the EDM parameters used in the one-output, two-input model. For the i^{th} element in $\boldsymbol{b}$, the nonstandardized *Beta* distribution is written as

$$f(b_i) = \frac{1}{U_i - L_i} \frac{\Gamma(\alpha_i + \beta_i)}{\Gamma(\alpha_i)\Gamma(\beta_i)} \left(\frac{b_i - L_i}{U_i - L_i}\right)^{\alpha_i - 1} \left(\frac{U_i - b_i}{U_i - L_i}\right)^{\beta_i - 1} \qquad (9.2)$$

for $L_i \le b_i \le U_i$ and $\alpha_i, \beta_i > 0$. For each of the parameter values b_i, its mean $\hat{\mu}_i$ is set at the assumed value from Chapter 5, its standard error is set to $\hat{\sigma}_i = CV\,\mu_i$, and the bounds L_i and U_i are set to $L_i = \hat{\mu}_i - 4\hat{\sigma}_i$ and $U_i = \hat{\mu}_i + 4\hat{\sigma}_i$. Then, L_i is bounded at 0 if b_i represents a supply elasticity and if $\hat{\mu}_i - 4\hat{\sigma}_i < 0$. Similarly, U_i is bounded at 0 if b_i represents a demand elasticity and if $\hat{\mu}_i + 4\hat{\sigma}_i > 0$. Given L_i and U_i, α_i and β_i are solved to obtain a mean of $\hat{\mu}_i$ and a standard deviation of $\hat{\sigma}_i$ for the distribution given in (9.2). It can be shown that setting $\alpha = \left(\frac{\phi_1}{\phi_1 + \phi_2}\right)(\phi_1\phi_2 - 1)$ and $\beta = \left(\frac{\phi_2}{\phi_1 + \phi_2}\right)(\phi_1\phi_2 - 1)$, where $\phi_1 = \left(\frac{\hat{\mu} - L}{\hat{\sigma}}\right)$ and $\phi_2 = \left(\frac{U - \hat{\mu}}{\hat{\sigma}}\right)$, results in a mean of $\hat{\mu}$ and a standard deviation of $\hat{\sigma}$.

15 Appendix 9A contains a more detailed discussion of the example contained in the textbook. The actual procedures are presented in the Excel workbook entitled "EDM examples Chapter 9.xlsm", which contains example data, the macros used to construct the simulated set of possible EDM metrics, and computations of confidence intervals for each metric.

16 We use the nonstandard *Beta* distribution because of its tractability. Researchers can use other estimated or literature-based marginal distributions such as triangular distributions or ranges of elasticity estimates.

If L_i and U_i are both 4 standard deviations from the mean, the marginal distribution is symmetric with $\alpha_i = \beta_i = 7.5$. Given L_i, U_i, α_i, and β_i, the BETAINV function in Excel was used to generate 1,000 b_i realizations corresponding to the quantiles (0.001, 0.002, …, 1) for each of the model's four elasticity point estimates. The resulting marginal b_i realizations were stacked into a 1,000 × 4 matrix, $\boldsymbol{Y}$. Parametric correlations were introduced using the Iman-Conover process, which reorders the elements in $\boldsymbol{Y}$ to obtain $\boldsymbol{Y}_C$. Each column in $\boldsymbol{Y}_C$ has the same rank order as a corresponding column in a 1,000 × 4 multivariate normal sample, $\boldsymbol{N}_C$**,** generated with covariance matrix $\hat{\Sigma}_b$ or its corresponding correlation matrix $\boldsymbol{R}_b$. The Iman-Conover process used here is equivalent to introducing marginal dependence using copula procedures with a 4-dimension normal or elliptical copula. By construction, the reordered matrix, $\boldsymbol{Y}_C$, has the same Spearman rank correlation matrix as $\boldsymbol{N}_C$. The Pearson correlations of $\boldsymbol{Y}_C$ obtained using reordered *Beta* distributions are similar to the model's specified correlation matrix, $\boldsymbol{R}_b$.

For each of the $j = 1, \dots 1{,}000$ joint $\boldsymbol{b}_j$ parameter realizations in $\boldsymbol{Y}_C$, the resulting EDM estimates proportional price and quantity changes, $E(\cdot)_j$, and surplus values CS_j, PS_j, $PS_{1,j}$, $PS_{2,j}$. These joint realizations are generated and stored. The set of $j = 1, \dots 1{,}000$ jointly simulated values — $E(\cdot)_j$, CS_j, PS_j, $PS_{1,j}$, $PS_{2,j}$ — are then used to construct empirical confidence intervals and to examine the joint structure between the EDM results.

A Numerical Example with Uncorrelated Elasticities

Appendix 9A presents the procedures used in this book's Excel workbook to generate confidence intervals for changes in EDM endogenous variables and surplus measures. For the first example, the standard deviation for each of the four elasticity estimates is calculated using an assumed coefficient of variation of 0.20. Of course, if actual standard errors for the elasticity estimates are available, they should be used. In addition, we initially assume zero correlations among the four elasticities. The results are presented in Table 9.1. The initial estimates are identical to those calculated in Chapter 5 for changes in the endogenous variables caused by a 10% excise tax on output. Changes in consumer and producer surplus for the tax increase are calculated in Chapter 8 and replicated in Table 9.1. The estimated EDM results for each of 1,000 Monte Carlo repetitions were calculated and stored using procedures discussed in Appendix 9A. The 1,000 simulated EDM results were used to generate the means and standard errors for each variable in Table 9.1. Note that the mean values from the simulations are very close to the initial estimates, which provides a check on the simulation procedures. In addition, the last two columns of Table 9.1 present the lower and upper bounds for a 95% confidence interval.

For the percentage change in output, $E(q)$, the EDM estimated that the excise tax would reduce output by 3.2%. The Monte Carlo simulations provide a mean

Table 9.1. 95% Confidence Intervals for the Endogenous Variables and Surplus Measures with Zero Correlations among the Input Supply Elasticities

Variable / Metric	Initial Estimate (5.11)	Mean/Standard Error for Simulated Estimates	Lower 0.025 Bound Estimate	Upper 0.975 Bound Estimate
$E(q)$	−0.032	−0.031 (0.004)	−0.039	−0.023
$E(p^D)$	0.053	0.053 (0.006)	0.040	0.065
$E(p^S)$	−0.047	−0.047 (0.006)	−0.060	−0.035
$E(x_1)$	−0.013	−0.013 (0.003)	−0.018	−0.008
$E(x_2)$	−0.039	−0.039 (0.005)	−0.048	−0.028
$E(w_1)$	−0.066	−0.066 (0.008)	−0.082	−0.049
$E(w_2)$	−0.039	−0.040 (0.007)	−0.053	−0.028
Consumer Surplus	−$51.80	−$51.75 (6.378)	−$64.56	−$39.66
Producer Surplus	−$46.64	−$46.70 (6.315)	−$58.87	−$34.17
Producer Surplus Input 1	−$19.61	−$19.63 (2.518)	−$24.44	−$14.64
Producer Surplus Input 2	−$27.09	−$27.14 (4.500)	−$36.70	−$19.17

estimate of −3.1% with a standard error of 0.004. Further, the lower bound of the 95% confidence interval is −3.9% and its upper bound is −2.3%. Hence, one would conclude that the EDM estimate for this endogenous variable is statistically different from 0 at the $\alpha = 0.05$ level. The other endogenous variables have similar interpretations.

The original EDM results indicated that the excise tax on output caused consumer surplus to decline by $51.80 using a market size measured by total revenue of $1,000 at the initial price and quantity equilibrium. The simulation results have a mean estimated decline in consumer surplus of $51.75 with an estimated standard deviation of $6.378, and 95% of the time lies within the interval between $64.56 and $39.66.

A Numerical Example with Correlated Elasticities

Many economic sectors contain correlated elasticity estimates. Brester, Marsh, and Atwood (2009) develop an EDM with vertical relationships within the meat sector in which supply elasticities at one level of the marketing chain, such as feeder cattle, are correlated with those at another level, such as fed cattle. In addition, elasticity estimates may be correlated among horizontally related markets. If an EDM contains elasticities of demand for both retail beef and retail pork, then it is likely that the two elasticities are correlated given that the two products are consumption substitutes. Finally, it could be that seemingly unrelated input supply elasticities could be correlated. For example, suppose an input (e.g., labor) is included in an

Table 9.2. 95% Confidence Intervals for the Endogenous Variables and Surplus Measures with Correlated Input Supply Elasticities

Variable / Metric	Initial Estimate (5.11)	Mean/Standard Error for Simulated Estimates	Lower 0.025 Bound Estimate	Upper 0.975 Bound Estimate
$E(q)$	−0.032	−0.031 (0.004)	−0.039	−0.022
$E(p^D)$	0.053	0.053 (0.007)	0.040	0.065
$E(p^S)$	−0.047	−0.047 (0.007)	−0.060	−0.035
$E(x_1)$	−0.013	−0.013 (0.003)	−0.018	−0.008
$E(x_2)$	−0.039	−0.039 (0.005)	−0.049	−0.028
$E(w_1)$	−0.066	−0.066 (0.009)	−0.082	−0.049
$E(w_2)$	−0.039	−0.040 (0.007)	−0.053	−0.028
Consumer Surplus	−$51.80	−$51.73 (6.646)	−$63.97	−$39.28
Producer Surplus	−$46.64	−$46.73 (6.608)	−$59.18	−$34.56
Producer Surplus Input 1	−$19.61	−$19.65 (2.579)	−$24.49	−$14.52
Producer Surplus Input 2	−$27.09	−$27.15 (4.494)	−$36.24	−$19.15

EDM along with a second input (e.g., capital). While it would not initially appear that the elasticity of labor supply would be related to the elasticity of capital supply, the elasticities could be correlated if an event (e.g., changes in information technologies) affected the supply responses of both inputs.

Therefore, we present procedures to estimate confidence intervals for the EDM described above using variances of each elasticity estimate and assume that the estimated own-price elasticities of supply of the two inputs have a correlation coefficient of 0.75. It is possible to empirically estimate these values. Alternatively, a researcher could use various estimates of such correlations and investigate whether they have a substantial effect on confidence intervals. Appendix 9A is used to obtain the estimates presented in Table 9.2. For this example, the assumed correlation has relatively minor impacts on the mean estimates and the ranges of the confidence intervals relative to the uncorrelated results presented in Table 9.1.

Summary

An EDM generates point estimates of changes in endogenous variables and surplus measures that are conditional upon a selected set of demand, supply, factor shares, and input substitution elasticities. Davis and Espinoza (2000) note that EDMs can be used to provide confidence intervals for those point estimates. One approach for evaluating the sensitivity of EDM results is to systematically vary a parameter value and record changes in the EDM results (Brester and Wohlgenant, 1997). However, if the desire is to estimate confidence intervals or perform hypothesis tests on EDM results, then potential joint realizations of an EDM's underlying parameters need to be considered (Davis and Espinoza, 2000). In addition, both the variability and potential dependency as measured by covariances or correlations among EDM parameters should be evaluated (Brester, Marsh, and Atwood, 2009).

With traditional multivariate methodologies, a researcher assumes a multivariate distribution — typically multivariate normal — and jointly estimates or assumes the moments of the multivariate parameters' distributions. It may be possible to simulate EDM results using variances and covariances obtained by the econometric estimation of systems of equations. In either case, copula procedures can be used to develop confidence intervals and examine the sensitivity of EDM results to potential joint variability in EDM model parameters.

»Chapter Nine, Appendix A

CONSTRUCTING AND ESTIMATING EDM CONFIDENCE INTERVALS USING SIMULATION PROCEDURES

This appendix presents numerical procedures that allow readers to estimate confidence intervals or perform hypothesis tests for EDM results, including price and quantity estimates and changes in consumer or producer surplus. The appendix presents a technical discussion of the procedures discussed in Chapter 9 and assumes that readers have access to the textbook's Excel workbook. We demonstrate how Excel can be used to complete all procedures while recognizing that other software programs can also be used. The procedure involves three components or stages:

Stage I: Generating a set of correlated joint EDM parameters.
Stage II: Computing and storing EDM results for joint parameter estimates.
Stage III: Computing confidence intervals using the simulated values from Stage II.

Stage I: Generating a Set of Correlated Joint EDM Parameters

The process used in Stage I consists of several steps:

Construct Column-Sorted Matrix of Possible Parameter Values

Table 9A.1 presents the method for constructing parameter marginals. In Chapter 9, we assumed that the set of EDM parameters η^D, ε_1, ε_2, and σ_{12} was jointly distributed with nonstandardized *Beta*(α, β, L, U) marginals and expected values (–0.60, 0.20, 1.0, and 1.0). For the example, the standard deviation of each of the four parameters (cells B5:E5) was set at 20% (cell B1) of the mean, L and U (cells B6:E7) were set 4 standard deviations from the mean and truncated if necessary, ϕ_L and ϕ_U (cells B9:E10) were computed as $\phi_L = \frac{\mu - L}{\sigma}$ and $\phi_U = \frac{U - \mu}{\sigma}$, and α and β (cells B11:E12) were computed as $\alpha = \left(\frac{\phi_L}{\phi_L + \phi_U}\right)(\phi_L\,\phi_U - 1)$ and $\beta = \left(\frac{\phi_U}{\phi_L + \phi_U}\right)(\phi_L \phi_U - 1)$, giving the set of nonstandardized *Beta* distribution parameters for each of the EDM variables (cells B6:E7 and B11:E12).[17]

We next construct a column-sorted, 1,000 × 4 matrix of possible (η^D, ε_1, ε_2, and σ_{12}) values by creating an index equal to 1:sample size or

17 Because we assume that the coefficient of variation = 0.20 and that L and U are 4 standard deviations from the mean, we did not need to truncate L or U in this example. Readers are encouraged to experiment with different L and U values while requiring negative demand and positive supply elasticities. The nonstandardized *Beta* distribution allows for positive or negative skewness in the marginal distributions if so desired.

Table 9A.1. Simulated Parameter Values

	A	B	C	D	E
1	cv =	0.2			
2					
3		η^D	ε_1	ε_2	σ_{12}
4	mean	-0.60	0.20	1.0	1.00
5	sdev	0.12	0.04	0.2	0.2
6	L-Bound (L)	-1.08	0.04	0.2	0.2
7	U-Bound (U)	-0.12	0.36	1.8	1.8
8					
9	θ_L	4	4	4	4
10	θ_U	4	4	4	4
11	α	7.5	7.5	7.5	7.5
12	β	7.5	7.5	7.5	7.5
13					
14	average	-0.6	0.2	1	1
15	sdev	0.119554	0.039851	0.199257	0.199257
16			sorted simulated values		
17	index	η^D	ε_1	ε_2	σ_{12}
18	1	-0.93312	0.088959	0.444795	0.444795
19	2	-0.91642	0.094526	0.472629	0.472629
20	3	-0.9056	0.098135	0.490673	0.490673
21	4	-0.89738	0.100873	0.504367	0.504367
22	5	-0.89067	0.103109	1.515544	1.515544

1:1000 in cells A18:A1017.[18] We use Excel's BETAINV function to construct possible parameter values. For example, in cell B18, the value −0.93312 = BETAINV($A18/1001,B$11,B$12,B$6,B$7) uses the index to compute a probability of (index-value/1001) while pulling the other BETAINV parameter values (α, β, L, U) from cells B11, B12, B6, and B7, respectively. By anchoring the appropriate rows and columns, we copy and paste the formula in cell B18 to cells B18:E1017 while allowing the BETAINV function's probability values to be dynamically adjusted from 1/1001 to 1000/1001.

Upon constructing the 1,000 possible parameter observations for each of the four parameters, we compute the mean and standard deviation of the simulated values (cells B14:E15) and note that these values closely match the specified values in cells B4:E5.

Construct a Cholesky Decomposition Matrix

In the following sections, we discuss the Chapter 9 example with an assumed correlation of 0.75 between the input supply elasticities ε_1 and ε_2. Table 9A.2 presents the correlation matrix $\boldsymbol{R}$ (cells H4:K7) and the resulting Cholesky decomposition matrix $\boldsymbol{C}$ (cells H10:K13) such that $\boldsymbol{C'C} = \boldsymbol{R}$. To compute the Cholesky decomposi-

18 An analyst can use any desired procedure to construct the sorted matrix (e.g., independently simulating vectors of possible outcomes for each parameter and then placing the sorted vectors into the matrix). We accomplish this in one step given that we assume nonstandardized *Beta* marginal distributions for all parameters.

Table 9A.2. The Correlation Matrix and Cholesky Decomposition Matrix

	G	H	I	J	K	L	M	N	O	P	Q
2											
3		η^D	ε_1	ε_2	σ_{12}			TEST	***C′C***		
4	η^D	1	0	0	0		1	0	0	0	
5	ε_1	0	1	0.75	0		0	1	0.75	0	
6	ε_2	0	0.75	1	0		0	0.75	1	0	
7	σ_{12}	0	0	0	1		0	0	0	1	
8											
9			CHOL	MATRIX C				TEST	***C′***		
10		1	0	0	0		1	0	0	0	
11		0	1	0.75	0		0	1	0	0	
12		0	0	0.6614378	0		0	0.75	0.661438	0	
13		0	0	0	1		0	0	0	1	
14											

tion matrix in Excel, we have added a Visual Basic Application (VBA) function, CHOL, to the example spreadsheet.[19] The CHOL function works similarly to Excel's other matrix functions in that, given a correlation or covariance matrix, a user must highlight a section of the spreadsheet of the same dimension as the correlation or covariance matrix, type =CHOL(select the correlation matrix range), and then simultaneously press Ctrl+Shift+Enter. In this example we highlighted the area H10:K13, typed CHOL(H4:K7) and then simultaneously pressed Ctrl+-Shift+Enter. The spreadsheet demonstrates that the resulting ***C*** matrix satisfies ***C′C*** by transposing the ***C*** matrix to ***C′*** (cells M10:P13) and multiplying ***C′C*** (cells M4:P7). Note that the values in M4:P7 equal those of the original correlation matrix (cells H4:K7).

Construct Matrices of Independent and Correlated Normal(0,1) Variates

Using the Data/Data Analysis tab, we generate matrix ***ZI*** by creating four columns of 1,000 independent Normal(0,1) variates in cells H18:K1017. We generate a matrix of correlated Normal(0,1) variates (***ZC*** = ***ZI*** × ***C***) by highlighting cells M18:P1017, typing =MMULT(H18:K1017,H10:K12) and pressing Ctrl+Shift+Enter.

Construct a Matrix of the Column Ranks of ZC Matrix

We construct a matrix containing the column ranks of the corresponding elements in the ***ZC*** matrix. We slightly modify Excel's rank command to accommodate potential equalities if two or more values in a column happen to have the same value. Consider the rank of the value –1.2763 in cell M18 of the ***ZC*** matrix. In cell R18 we type: = RANK(M18,M$18:M$1017,1) + COUNTIF(M$18:M18,M18)–1. Given the cell anchoring in this formula, we copy this formula into cells

19 We have found that the CHOL function values in cells H10:K13 do not always compute correctly when the spreadsheet is opened. For these cases, users need to retype the CHOL function and press Ctrl+Shift+Enter as described above to construct the correct Cholesky matrix, **C**.

R18:U1017 and the values in cells R18:U1017 contain the column ranks of the corresponding ***ZC*** elements. For example, the value −1.2763 in cell M18 has rank 105 (in cell R18) indicating that −1.2763 is the 105th lowest of the values in cells M18:M1017. Similarly, the rank of 0.19917 in cell N18 (relative to the values in cells N18:N1017) is 551.

In this example, we arbitrarily specified a correlation of 0.75 between ε_1 and ε_2, implying that, on average, below-average values of ε_1 should be associated below-average values of ε_2 and vice versa. This should be reflected in the rank matrix. An examination of values in cells S18:S1017 and T18:T1017 indicates that the corresponding sets of ranks follow the desired pattern. A scatterplot of the corresponding ranks presented in Figure 9A.1 demonstrates that the desired rank results resemble a normal copula plot when constructed with correlation 0.75.

Construct a Matrix of Correlated EDM Parameter Values

To introduce correlation between the simulated EDM parameter values, we use the ranks in R18:U1017 to place the potential parameter values in cells B18:E1017 in the same column-rank order as the columns of the ***ZC*** matrix. This is accomplished using Excel's LOOKUP function. The formula in cell X18 is =LOOKUP(R18,$A$18:$A$1017,B$18:B$1017), which looks up the R18 value of 105 in the index values of index cells A18:A1017 and pulls the 105th lowest potential η^D value of −0.75383 from the 105th element in cells B18:B1017 and places the value −0.75383 in cell X18. The anchored formula in cell X18 is copied into cells X18:AA1017, giving a matrix of correlated potential joint EDM parameters ($\eta^D, \varepsilon_1, \varepsilon_2, \sigma_{12}$).

Cell Z12 computes the sample correlation between the 1,000 simulated ε_1 and ε_2 values as 0.722, which differs from the specified value of 0.75 only because of sampling errors that occur when constructing the ***ZI*** matrix. Figure 9A.2 plots the 1,000 ε_1 and ε_2 values and illustrates their positive correlation.

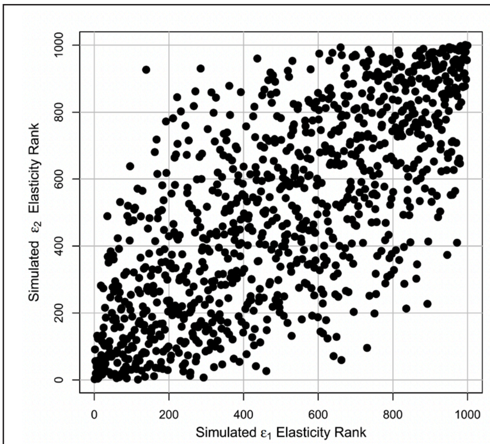

Figure 9A.1. Input Supply Elasticities Rank Plot

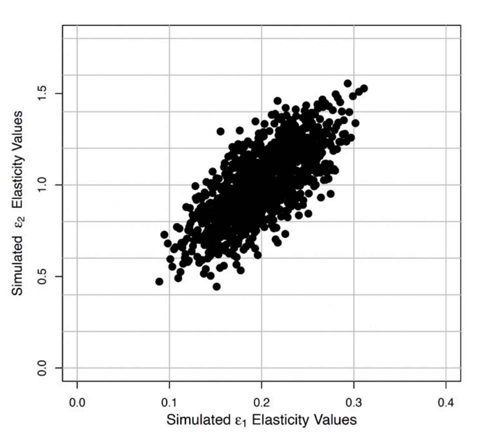

Figure 9A.2. Simulated Correlations of Input Supply Elasticities

We conclude this section by noting that the columns of values in cells X18:AA1017 are distributed as nonstandardized *Beta* variates but have been reordered and bound together with a normal copula via the Iman-Conover rank-ordering process. The procedures allow a researcher to independently specify, assume, or estimate any functional or empirical distribution for each parameter while introducing dependency using a normal copula. Similar rank ordering procedures can be used to introduce dependency using other copulas and replacing the ***ZC*** matrix with simulated joint copula realizations.

Stage II: Computing and Storing EDM Results for Joint Parameter Estimates

We use Excel's VBA macros to implement an iterative process to compute and store the EDM results from each of the 1,000 joint parameter realizations constructed in Stage I in the EDM tab. We modified the EDM model (cells F6, F7, F8, and F9) to use a set of EDM input parameters from cells B95:E95 and place this input vector's associated EDM output results in cells J95:T95. Macros are used to iteratively place each of the 1,000 sets of EDM input parameters created in Stage I into cells B95:E95 and then copy and store the input vector and its associated EDM outputs. Upon completion of the process, the spreadsheet has 1,000 sets of input parameters and EDM results.

The spreadsheet uses three Excel macros to compute and store the EDM simulations results: ***reset***, ***nextone***, and ***go***. The ***reset*** macro is initiated by typing Ctrl+R. The macro: (1) erases all previously computed sets of EDM input-output results from the EDM_Model tab, (2) copies the current set of potential joint input parameters from the parameters tab into the EDM_Model tab, and (3) initializes several sets of cells in the EDM_Model tab. Table 9A.3 demonstrates the structure of the first nine rows in the input-output section of the EDM_Model tab after the ***reset*** macro has been executed.

The macro ***nextone*** (Ctrl+N): (1) copies the index number and the vector of input parameters from the bottom the input parameters matrix (row 1100) and

Table 9A.3. First Nine Rows of the Input-Output Section of EDM_Model Tab

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
94	index	η^D	ε_1	ε_2	σ_{12}					$E(q)$	$E(p^D)$	$E(p^S)$	$E(x_1)$	$E(x_2)$	$E(w_1)$	$E(w_2)$	CS	PS	$PS1$	$PS2$
95	0	-0.6	0.2	1	1					-0.03158	0.052631579	-0.04737	-0.01316	-0.0394737	-0.06579	-0.0394737	-$51.80	-$46.62	-$19.61	-$27.09
96																				
97																				
98																				
99	index	η^D	ε_1	ε_2	σ_{12}					$E(q)$	$E(p^D)$	$E(p^S)$	$E(x_1)$	$E(x_2)$	$E(w_1)$	$E(w_2)$	CS	PS	$PS1$	$PS2$
100	0	-0.6	0.2	1	1															
101	1	-0.753832	0.20532471	0.975494219	0.79616652															
102	2	-0.637244	0.178988263	0.854085235	1.03885412															
103	3	-0.519407	0.14719966	0.861857309	0.82451534															
104	4	-0.649564	0.194992999	1.000262968	0.7684123															
105	5	-0.59258	0.213292339	0.76275639	0.7133261															
106	6	-0.636588	0.248614762	1.1068288	0.70266985															
107	7	-0.539427	0.243450308	1.285394381	0.93737953															
108	8	-0.573798	0.219030243	1.166924788	1.11398055															

pastes the values into the EDM input cells A91:E91, (2) inserts a new row in row 100 to hold the most recent vector of input-output results, (3) copies the current input-output values from row 95 into row 100, (4) erases the index and input values from the bottom of the input matrix (now row 1101), and (5) returns the cursor to cell A90.[20] The net effect is that, when run repeatedly, the ***nextone*** macro processes simulated potential input outcomes as set 1000, then 999, then 998, …., then 1, and then set 0 which is the original assumed parameters. The final results are indexed order 0, 1, 2, …, 1000 in cells A100:T1100. Table 9A.4 demonstrates the structure of the first nine rows in the input-output section of the EDM_Model tab.

The macro ***go*** (Ctrl+G): (1) invokes the ***reset*** macro, (2) executes a loop that invokes the ***nextone*** macro 1,001 times, and (3) executes the ***summary*** macro described in the next section.

Table 9A.4. First Nine Populated Rows of the Input-Output Section of EDM_Model Tab

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
94	index	η^D	ε_1	ε_2	σ_{12}					$E(q)$	$E(p^D)$	$E(p^S)$	$E(x_1)$	$E(x_2)$	$E(w_1)$	$E(w_2)$	*CS*	*PS*	*PS1*	*PS2*
95	0	-0.6	0.2	1	1					-0.03158	0.052631579	-0.04737	-0.01316	-0.0394737	-0.06579	-0.0394737	-$51.80	-$46.62	-$19.61	-$27.09
96																				
97																				
98																				
99	index	η^D	ε_1	ε_2	σ_{12}					$E(q)$	$E(p^D)$	$E(p^S)$	$E(x_1)$	$E(x_2)$	$E(w_1)$	$E(w_2)$	*CS*	*PS*	*PS1*	*PS2*
100	0	-0.6	0.2	1	1					-0.03158	0.052631579	-0.04737	-0.01316	-0.0394737	-0.06579	-0.0394737	-51.8006	-46.62049861	-19.607	-27.0862
101	1	-0.753832	0.20532471	0.975494219	0.79616652					-0.03471	-0.046047437	-0.05395	-0.01592	-0.0427644	-0.07755	-0.0438387	-45.2482	-53.01616077	-23.0803	-30.0309
102	2	-0.637244	0.178988263	0.854085235	1.03885412					-0.03048	-0.047827494	-0.05217	-0.01245	-0.038206	-0.06953	-0.0447333	-47.0987	-51.37745488	-20.7294	-30.7151
103	3	-0.519407	0.14719966	0.861857309	0.82451534					-0.02688	-0.051747791	-0.04825	-0.0101	-0.0340695	-0.0686	-0.0395303	-51.0523	-47.6037438	-20.4771	-27.1998
104	4	-0.649564	0.194992999	1.000262968	0.7684123					-0.03238	-0.04984973	-0.05015	-0.01435	-0.0401065	-0.07361	-0.0400959	-49.0426	-49.3383225	-21.9246	-27.5043
105	5	-0.59258	0.213292339	0.76275639	0.7133261					-0.02825	-0.047670811	-0.05233	-0.01509	-0.0338863	-0.07077	-0.0444261	-46.9975	-51.59007083	-21.0707	-30.5714
106	6	-0.636588	0.248614762	1.1068288	0.70266985					-0.03382	-0.043122863	-0.04688	-0.01745	-0.0408334	-0.07018	-0.0368923	-52.2246	-46.08450639	-20.8689	-25.2973
107	7	-0.539427	0.243450308	1.285394381	0.93737953					-0.03254	-0.0603239	-0.03968	-0.01438	-0.0403248	-0.05905	-0.0313716	-59.3424	-39.03056297	-17.5886	-21.5173
108	8	-0.573798	0.219030243	1.166924788	1.11398055					-0.0328	-0.057169447	-0.04283	-0.01323	-0.0411925	-0.0604	-0.0353001	56.2318	-42.12805257	-18.0006	-24.2001

Stage III: Computing Confidence Intervals Using the Simulated Values from Stage II.

After completing Stages I and II, we now have an array containing 1,000 simulated EDM outcomes and the original EDM outcomes in cells J100:T1100. The simulated outcomes in cells J101:T1100 can be used to perform hypothesis tests or develop confidence intervals. We present the simulated means, standard deviations, and confidence intervals in cells H89:T92 using the macro ***summary*** (Ctrl+S) which is invoked at the end of the ***go*** macro described above.

20 We used the macro title ***nextone*** because the expression ***next*** is a reserved VBA term.

The user can specify a desired two-tailed confidence interval in cell G92 and the confidence interval formulas in cells J91:T92 automatically compute the confidence intervals. Table 9A.5 presents the resulting summary table.

Table 9A.5. Completed Input-Output Section of EDM_Model Tab

	H	I	J	K	L	M	N	O	P	Q	R	S	T
86				**SUMMARY STATISTICS AND CONFIDENCE INTERVALS**									
87													
88			$E(q)$	$E(p^D)$	$E(p^S)$	$E(x_1)$	$E(x_2)$	$E(w_1)$	$E(w_2)$	*CS*	*PS*	*PS1*	*PS2*
89		mean	-0.031	0.053	-0.047	-0.013	-0.039	-0.066	-0.040	-51.728	-46.726	-19.649	-27.149
90		sdev	0.004	0.007	0.007	0.003	0.005	0.009	0.007	6.646	6.608	2.579	4.494
91	CI_UP	0.975	-0.022	0.065	-0.035	-0.008	-0.028	-0.049	-0.028	-39.281	-34.558	-14.520	-19.150
9.2	CI_LO	0.025	-0.039	0.040	-0.060	-0.018	-0.049	-0.082	-0.053	-63.969	-59.182	-24.486	-36.247

»Chapter Ten

SUMMARY

To our knowledge, this is the first publication to comprehensively present the theory and development of equilibrium displacement models (EDMs). Components of this effort can be found scattered among mimeographs, working papers, reports, class notes, and often only briefly discussed in journal articles because of space limitations. Some previously developed EDMs contain errors and mistakes, which is a common occurrence when most learn of these models from classroom assignments. A careful look at the literature reveals a network of EDM activity centered around North Carolina State University's Department of Agricultural and Resource Economics and agricultural economists who were either faculty or doctoral students in that program. Many of these faculty and students continued to use EDMs in their research careers at universities and government agencies throughout the United States and the world.

What have we learned from writing this textbook? The agricultural economics literature is replete with EDMs used to examine various economic policies. EDMs can be valuable tools when a researcher is considering the impacts of exogenous shocks or government policies. However, because EDMs provide linear approximations of shifts or movements along demand and supply curves, only relatively small shocks should be considered as underlying demand and supply curves are not likely to be linear. Of course, larger shocks can be considered if the underlying demand and supply functions are assumed to be only modestly nonlinear. We also note that maintaining theoretical economic restrictions is crucial for generating unbiased price, quantity, and surplus results.

We also note that the data needs required for EDMs are not excessive but can be problematic depending upon the application. Specifically, the factor shares of each input used in a production process are needed, but such data are not always available. Many farm management record systems contain information on the total and variable costs of inputs such as crop nutrients, crop protectants, energy, seed, labor, and others. Similar data can sometimes be obtained for other firms using sources such as the U.S. Census of Manufactures.

Several elasticity estimates are needed when building EDMs. Own-price elasticities of demand are required. If substitutes or complements are included in an EDM, then cross-price elasticities of demand are also needed. In addition, own-price elasticities of supply for each input used in the production process are required. In many cases, most of these elasticity estimates can be obtained from the extant literature. However, if a researcher develops their own estimates, then variances and covariances of elasticity estimates are more readily available. These are useful for developing confidence intervals for EDM results and for hypothesis testing. In addition, many useful elasticity estimates can be obtained from EDM results (e.g., the own-price elasticity of output supply, own-price elasticities of input demand, etc.).

Allen elasticities of substitution (AES) among inputs used in a production process are needed to construct an EDM. In some cases, elasticities of substitution may be obtained from the literature. Although Mullen, Wohlgenant, and Farris (1988) describe a method for estimating these elasticities in the dual space, it is rare for researchers to have the data needed to obtain AES estimates in the primal space. Our experience is that the magnitude of AES estimates is less important than ensuring that nonzero values are included in EDMs. It is likely rare that fixed input proportion technologies are reasonable descriptions of production processes. If no other information is available, it seems reasonable to at least assume an AES of 1.0 as a starting point. It is relatively easy to evaluate the sensitivity of EDM results to ranges of AES estimates (Hamilton et al., 2020).

Perhaps as important as any of the above data needs, researchers must have a solid understanding of the industry being modeled. Decisions as whether to include vertical or horizontal linkages and the necessary inputs must be made. Further, one must carefully consider the impact of a shock or policy on market outcomes to develop appropriate price and quantity wedges and other constraints. We find that graphically representing the outcome of a shock or policy provides valuable guidance when altering an EDM to reflect those changes. We are again reminded of the oft-repeated phrase that "all we know about economics, we learned in ECON 101."

We have shown that some methodologies and studies, including some by these authors, suffer from a fundamental flaw regarding homogeneity of production processes. That is, EDM production technologies must be homogeneous of degree 0 (HD0) in all input and output prices. The consequences of this oversight are biased estimates of price, quantity, and surplus changes caused by exogenous economic or policy shocks. We also note that our development of a theoretically consistent EDM involves converting the primal problem of profit maximization into its dual structure. It is interesting to note that other researchers have developed theoretically consistent EDMs using different approaches. Once again, we were unable to find a published account of how these models were fully developed. We conclude that some of our colleagues, both past and present, had or have an ability to “see” what a theoretically consistent EDM specification should look like. Oh, to have that ability.

Policy makers should consider both the intended and unintended consequences that result from legislation. One aspect of these consequences can be measured by changes in producer and consumer surplus. The pioneering efforts of Just, Hueth, and Schmitz (2004) provide a careful roadmap for appropriately measuring these changes.

Although seldom done, it is often useful to evaluate the sensitivity of EDM results to underlying elasticity (and perhaps factor share) estimates. We have illustrated one method for estimating confidence intervals for these results that also allows for hypothesis testing. We note that other simulation methods are available.

The Farm Bill and policies that impact farmers, ranchers, food processors, and consumers continue to interest researchers because policy makers and stakeholder groups in these industries desire information on the impacts of legislation. Many of the intervention tools discussed in this textbook are no longer used in the United States to support domestic producers since the 1996 Federal Agriculture Improvement and Reform Act and other trade and policy reforms of the 1990s. Nonetheless, it seems that legislators often discuss returning to such policies to support domestic producers in various economic sectors. Governmental market interventions (e.g., price ceilings and floors, excise and sales taxes, tariffs, quotas, subsidies) continue to be implemented for various reasons. Such interventions affect agricultural supply and food value chains. As the agricultural economics profession evolves into applied research areas such as economic development, environment, food aid, nutrition, and natural resources, EDMs can be formulated to evaluate these policies. The USDA’s National Agricultural Statistics Service and Economic Research Service have been important sources of data for developing EDMs used to understand the impact of a wide variety of government policies. Additional data are being created through precision agriculture, scanners, and other activities that may allow for better estimates of elasticities and factor shares in the future.

We hope this textbook will be valuable to researchers who undertake policy analyses. In addition, we have found that EDMs are useful tools for teaching economic relationships among markets. The accompanying Excel workbooks should help those who want to use EDMs in their research and teaching activities.

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Applied economists use equilibrium displacement models (EDMs) for policy analyses. EDMs allow for the estimation of price and quantity changes resulting from exogenous economic or policy shocks across related markets. The models are also used to estimate changes in producer and consumer surplus caused by such shocks and to quantify their short- and long-term impacts. Because complex interactions exist in many markets, EDMs provide a comprehensive approach to modeling changes in market equilibria. This book uses a primal approach to formally develop EDMs in the dual space. In addition, a variety of applications are provided that allow researchers to develop EDMs for specific market and policy situations.